Theorem: If $f(x) \leq g(x)$ near a (except possibly at a) and if $lim_{x \to a} f(x)$ and $lim_{x \to a} g(x)$ exist, then

$$lim_{x \to a} f(x) \leq lim_{x \to a} g(x)$$

Defn: $lim_{x \to a} f(x) = L$ if

x close to a (except possibly at a) implies $f(x)$ is close to L.

Squeeze theorem:
If $f(x) \leq g(x) \leq h(x)$ near a (except possibly at a) and if $lim_{x \to a} f(x) = L$ and $lim_{x \to a} h(x) = L$, then

$$lim_{x \to a} g(x) = L$$

Example: $g(x) = x sin \frac{1}{x}$

Defn: $lim_{x \to a} f(x) = L$ if

x close to a (except possibly at a) implies $f(x)$ is close to L.

Defn: $lim_{x \to a} f(x) = L$ if
for all $\epsilon > 0$, there exists a $\delta > 0$ such that
$0 < |x - a| < \delta$ implies $|f(x) - L| < \epsilon$

Show $lim_{x \to 1} 2 =$

Defn: $lim_{x \to a} f(x) = L$ if
for all $\epsilon > 0$, there exists a $\delta > 0$ such that
$0 < |x - a| < \delta$ implies $|f(x) - L| < \epsilon$

Show $lim_{x \to 4} 2x + 3 =$

Defn: $lim_{x \to a^-} f(x) = L$ if

x close to a and $x < a$
implies $f(x)$ is close to L.

Defn: $lim_{x \to a^+} f(x) = L$ if

x close to a and $x > a$
implies $f(x)$ is close to L.

Defn: $lim_{x \to a} f(x) = \infty$ if

x close to a (except possibly at a)
implies $f(x)$ is large.

Defn: $lim_{x \to a} f(x) = -\infty$ if

x close to a (except possibly at a)
implies $f(x)$ is negative and $|f(x)|$ is large.