Theorem: If \( f(x) \leq g(x) \) near \( a \) (except possibly at \( a \)) and if \( \lim_{x\to a} f(x) \) and \( \lim_{x\to a} g(x) \) exist, then
\[
\lim_{x\to a} f(x) \leq \lim_{x\to a} g(x)
\]

Squeeze theorem:
If \( f(x) \leq g(x) \leq h(x) \) near \( a \) (except possibly at \( a \)) and if \( \lim_{x\to a} f(x) = L \) and \( \lim_{x\to a} h(x) = L \), then
\[
\lim_{x\to a} g(x) = L
\]

Example: \( g(x) = x \sin \frac{1}{x} \)
Defn: $\lim_{x \to a} f(x) = L$ if

$x$ close to $a$ (except possibly at $a$) implies $f(x)$ is close to $L$. 
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Defn: \( \lim_{x \to a} f(x) = L \) if
for all \( \epsilon > 0 \), there exists a \( \delta > 0 \) such that
\( 0 < |x - a| < \delta \) implies \( |f(x) - L| < \epsilon \)

Show \( \lim_{x \to 1} 2 = \)
Defn: \( \lim_{x \to a} f(x) = L \) if
for all \( \epsilon > 0 \), there exists a \( \delta > 0 \) such that
\( 0 < |x - a| < \delta \) implies \( |f(x) - L| < \epsilon \)

Show \( \lim_{x \to 4} 2x + 3 = \)
Defn: $\lim_{x \to a^-} f(x) = L$ if

$x$ close to $a$ and $x < a$

implies $f(x)$ is close to $L$.

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Defn: $\lim_{x \to a^+} f(x) = L$ if

$x$ close to $a$ and $x > a$

implies $f(x)$ is close to $L$. 
Defn: $\lim_{x \to a} f(x) = \infty$ if

$x$ close to $a$ (except possibly at $a$)
implies $f(x)$ is large.

Defn: $\lim_{x \to a} f(x) = -\infty$ if

$x$ close to $a$ (except possibly at $a$)
implies $f(x)$ is negative and $|f(x)|$ is large.