Other methods
(For example if you don't have continuous function on a closed interval)
(or finding 2nd derivative easy)

- analyze the graph

Special cases
1) $f''(x) < 0$ for all $X \in \mathbb{R}$

- abs max

- no abs max

- $f''$ does not exist at this point
Thm 9: If $f'(x)$ exists for all $x$ in interval $I$ and if $f'(c) = 0$ and $c$ is the only critical point in $I$, then:

$f''(c) < 0 \Rightarrow f(c) \text{ is a max}$

$f''(c) > 0 \Rightarrow f(c) \text{ is a min}$

Other methods for use: first derivative
3.5) Application of abs max/min

Ex 1) Find 2 numbers whose sum is 100 that have maximum product

1.) What are you maximizing?
product = \[ P = xy \]

2.) Turn into a Calc 1 problem by eliminating all but one variable by RHS of eqn

Need eqn that relates \( x \& y \)
to eliminate \( y \)

\[ x + y = 100 \]
\[ y = 100 - x \]
Calc 1 problem:

Maximize \( P(x) = x(100-x) \)

3) Solve Calc 1 problem

Note no restrictions on \( x \)

3a) Find critical pts

\[ P'(x) = 100 - 2x = 0, \ DNE \]

\[ 100 = 2x \]

\[ x = 50 \]

3b) Determine if abs max

i) Can't use EVT - no restrictions on \( x \), i.e. don't have closed interval

ii) \( P''(x) = -2 < 0 \)

By analyzing graph \[ \Rightarrow \]

abs max at \( x = 50 \)
iii) By Thm 9, we note
\[ p'(x) = 100 - 2x \] exists for all \( x \)
\[ p'(50) = 0 \]
So is the only critical point of \( p \)
\[ p''(50) = -2 < 0 \]
\[ \Rightarrow \text{ are max at } x = 50 \]
by Thm 9

4) State sol'n
\[ x = 50 \quad y = 100 - 50 = 50 \]
Two #’s are 50, 50
EX2) Suppose you have 200 m of fencing. What is the largest rectangular area you can fence in? What are the dimensions?

1) Draw picture

\[ w \quad \frac{l}{w} \]

I) What are we maximizing?

\[ A = l \cdot w \]

2) Eliminate variable via substitution

Need eqn relating \( l \) \& \( w \) so I can eliminate one of them

Perimeter = 200 = 2\( l \) + 2\( w \)
100 = l + w

l = 100 - w

A(w) = (100 - w) w

3) Maximize

A(w) = 100w - w^2

where w ∈ [0, 100]

3a) Find critical points

A'(w) = 100 - 2w

⇒ w = 50

3b) Determine if abs max at w = 50

i) EVT

<table>
<thead>
<tr>
<th>w</th>
<th>A(w) = (100 - w) w</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>50 * 50 = 2500</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>100</td>
<td>0</td>
</tr>
</tbody>
</table>

abs max occurs at x = 50

ii) f''(x) = -2 < 0 ∀ for all x

⇒ f'(50) = 0
4) State Answer

Largest area = 2500 m²

Dimensions: 50 m x 50 m

\[ x = 50 \]
\[ y = 100 - 50 = 50 \]