Find the following for $f(x) = \frac{x^2+3x}{x-1} = \frac{x(x+3)}{x-1}$ (if they exist; if they don't exist, state so). Use this information to graph f.

Note $f'(x) = \frac{(x-3)(x+1)}{(x-1)^2}$, $f''(x) = \frac{8}{(x-1)^3}$

[1.5] 1a.) critical numbers: $3, -1, \cancel{1}$

[1.5] 1b.) relative maximum(s) occur at $x =$ -1

[1.5] 1c.) relative minimum(s) occur at $x =$ 3

[1.5] 1d.) The absolute maximum of f on the interval [0, 5] is none and occurs at $x =$ ______

[1.5] 1e.) The absolute minimum of f on the interval [0, 5] is none and occurs at $x =$ ______

[1.5] 1f.) Inflection point(s) occur at $x =$ none

[1.5] 1g.) f increasing on the intervals $(-\infty, -1) \cup (3, \infty)$

[1.5] 1h.) f decreasing on the intervals $(-1, 1) \cup (1, 3)$

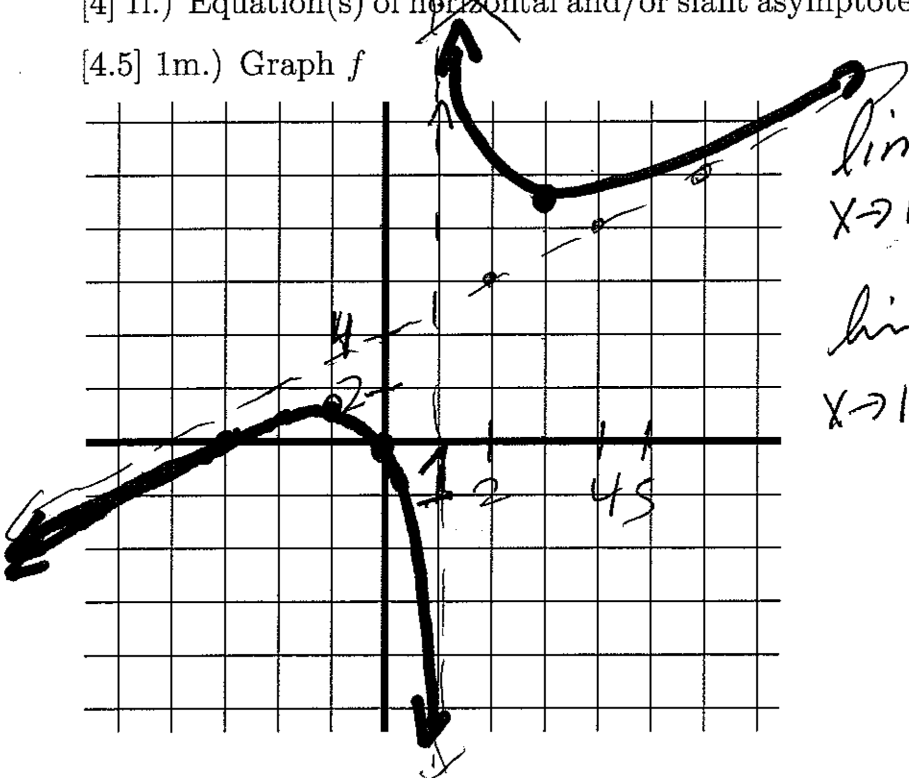
[1.5] 1i.) f is concave up on the intervals $(1, \infty)$

[1.5] 1j.) f is concave down on the intervals $(-\infty, 1)$

[1.5] 1k.) Equation(s) of vertical asymptote(s) $X = 1$

[4] 1l.) Equation(s) of ~~horizontal~~ and/or slant asymptote(s) $y = X + 4$

[4.5] 1m.) Graph f



$$\lim_{X \to 1^-} \frac{X(X+3)}{(X-1)} = -\infty$$

$$\lim_{X \to 1^+} \frac{X(X+3)}{(X-1)} = +\infty$$

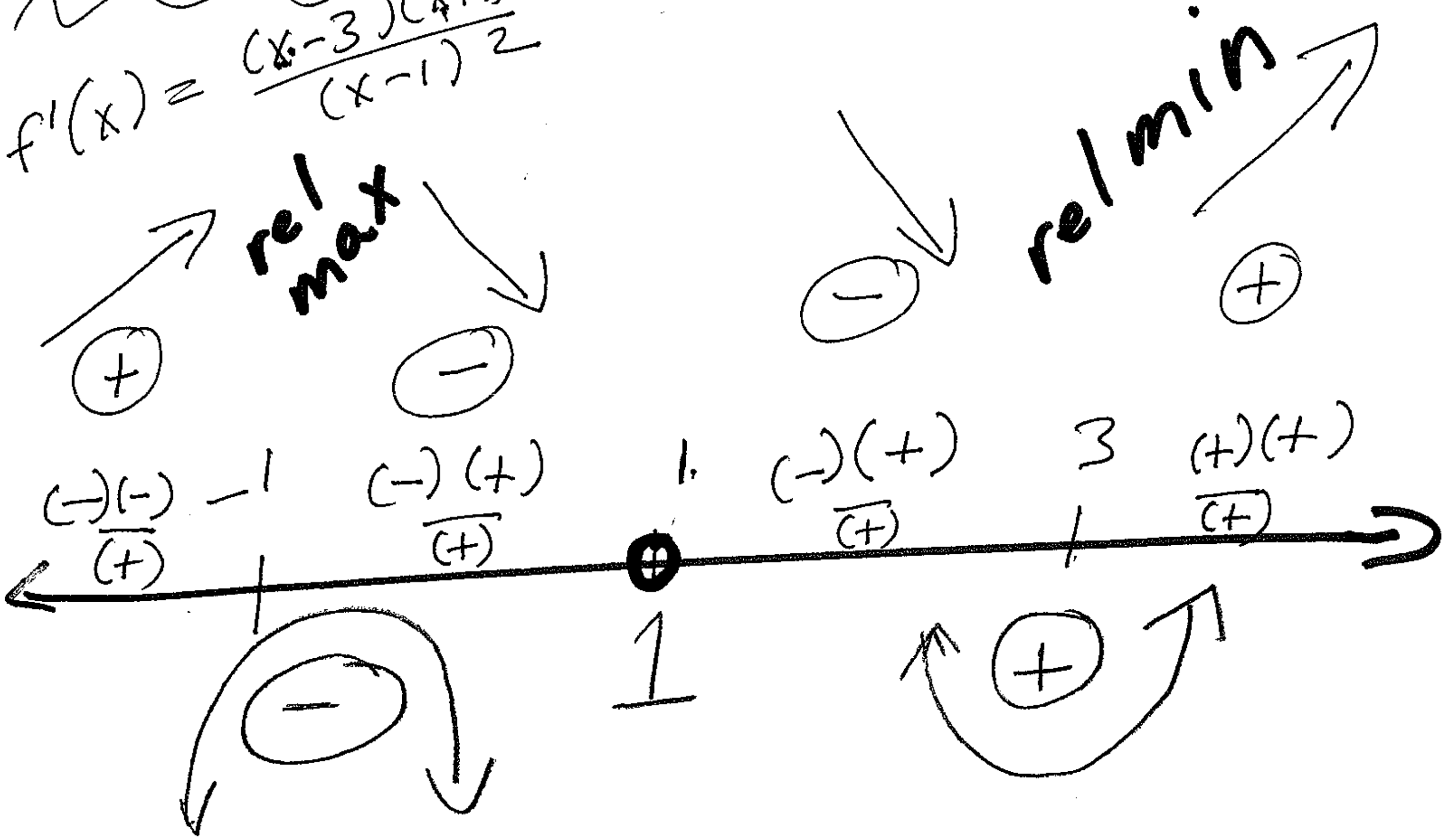
$$f'(x) = \frac{(x-3)(x+1)}{(x-1)^2} = 0, \text{DNE}$$

$$x = 3, -1, 1$$

$$f''(x) = \frac{8}{(x-1)^3} = 0, \text{DNE}$$

$$x = 1$$

$$f'(x) = \frac{(x-3)(x+1)}{(x-1)^2}$$



$$f''(x) = \frac{8}{(x-1)^3}$$

$$\lim_{x \to +\infty} \frac{x^{2}+3x}{x-1} = +\infty$$

$$\lim_{x \to -\infty} \frac{x^2+3x}{x-1} = -\infty$$

$$\frac{(+)}{(-)}$$

no horizontal asymptote
But we do have a slant asymptote

$$\begin{array}{r} x + 4 \ \text{r}\ 4 \\ (x-1)\overline{)\,x^2+3x} \\ \underline{x^2 - x} \\ 4x \\ \underline{4x-4} \\ +4 \end{array}$$

$$\frac{x^2+3x}{x-1}$$

$$= \underline{x+4} + \frac{4}{x-1} \to 0$$

$\approx x+4$ for large x

check : $$\frac{(x+4)(x-1)+4}{x-1}$$

$$= \frac{x^2+3x-4+4}{x-1}$$

$$\frac{x^2+3x}{x-1} = x+4+\underbrace{\frac{4}{x-1}}_{\text{small rounding error large large } x}$$

$$\sim x+4$$

for large x
(& medium valued x's too)

For really large x

$x+4 \sim x$ but we want better approx for slant asym

Slant asym

$$\boxed{y = x+4}$$

x	$y = \frac{(x^2+3x)}{x-1}$
-1	$(1-3)/-2 = 1$
3	$18/2 = 9$
0	0
-3	0

$$\frac{x^2+3x}{\cancel{x-1}} = 0$$

$$x(x+3) = 0$$

$\boxed{x \neq 2, -2}$

Find the following for $f(x) = \frac{x^2}{x^2-4} = \frac{x^2}{(x+2)(x-2)}$ (if they exist; if they don't exist, state so). Use this information to graph f.

since potential pts were NOT in domain

Note $f'(x) = \frac{-8x}{(x^2-4)^2}$, $f''(x) = \frac{8(3x^2+4)}{(x^2-4)^3}$

f' [1.5] 1a.) critical numbers: 0, 2, -2

f'' f' [1.5] 1b.) relative maximum(s) occur at $x =$ 0

f'' f' [1.5] 1c.) relative minimum(s) occur at $x =$ none

[1.5] 1d.) The absolute maximum of f on the interval [0, 5] is none and occurs at $x =$ ______

[1.5] 1e.) The absolute minimum of f on the interval [0, 5] is none and occurs at $x =$ ______

f'' [1.5] 1f.) Inflection point(s) occur at $x =$ none

f' [1.5] 1g.) f increasing on the intervals $(-\infty,-2) \cup (-2,0)$

$x<-2$

$-2<x<0$

f' [1.5] 1h.) f decreasing on the intervals $(0,2) \cup (2,\infty)$

f'' [1.5] 1i.) f is concave up on the intervals $(-\infty,-2) \cup (2,\infty)$

f'' [1.5] 1j.) f is concave down on the intervals $(-2,2)$

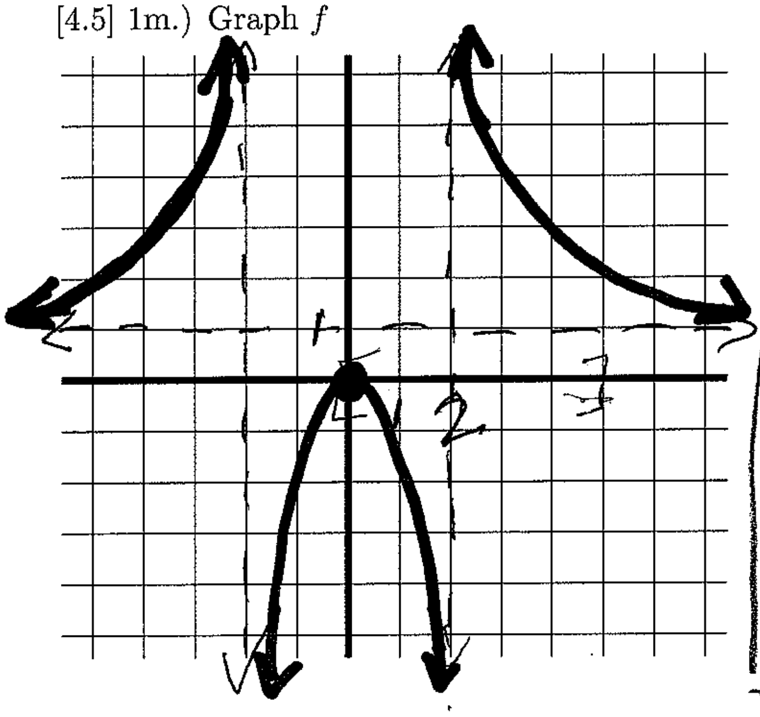
[1.5] 1k.) Equation(s) of vertical asymptote(s) $x=2,\ x=-2$

[4] 1l.) Equation(s) of horizontal and/or slant asymptote(s) $y=1$

$$\frac{x^2}{x^2-4} \sim \frac{x^2}{x^2} = 1$$

for large x

[4.5] 1m.) Graph f



$\lim_{x\to 2^+}$ $\lim_{x\to 2^+}$ $\ominus$

$\lim_{x\to 2^-}$ $\lim_{x\to 2^-}$ $\ominus$

$5\sqrt[3]{4} - \sqrt[3]{2^5} = 5\sqrt[3]{4} - 2\sqrt[3]{4} = 3\sqrt[3]{4}$

Find the following for $f(x) = 5x^{\frac{2}{3}} - x^{\frac{5}{3}}$ (if they exist; if they don't exist, state so). Use this information to graph f.

$f'(x) = 0, DNE$

f' [1.5] 1a.) critical numbers: $0, 2$

f'' f' [1.5] 1b.) relative maximum(s) occur at $x =$ 2

f'' f' [1.5] 1c.) relative minimum(s) occur at $x =$ 0

[1.5] 1d.) The absolute maximum of f on the interval [0, 5] is $3\sqrt[3]{4}$ and occurs at $x =$ 2

[1.5] 1e.) The absolute minimum of f on the interval [0, 5] is 0 and occurs at $x =$ $0, 5$

f'' [1.5] 1f.) Inflection point(s) occur at $x =$ -1

f' [1.5] 1g.) f increasing on the intervals $(0, 2)$ ← $0 < x < 2$

f' [1.5] 1h.) f decreasing on the intervals $(-\infty, 0) \cup (2, \infty) = \{x < 0 \text{ or } x > 2\}$

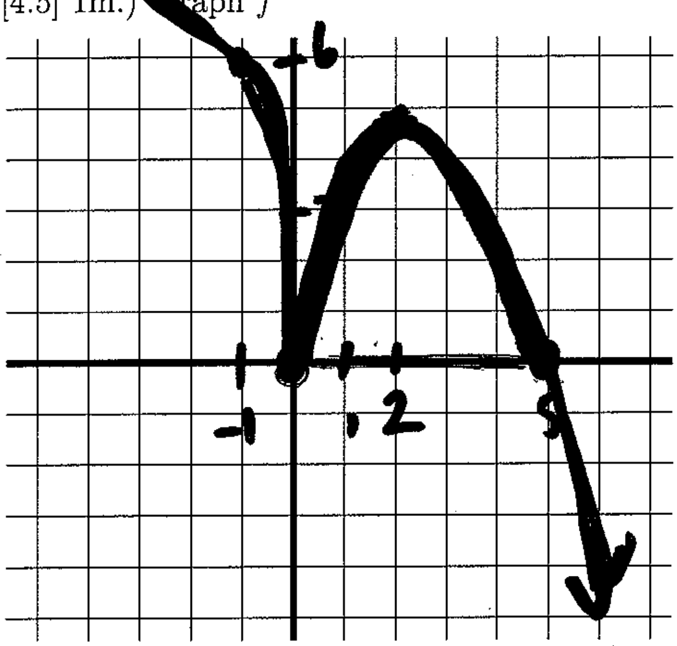
f'' [1.5] 1i.) f is concave up on the intervals $(-\infty, -1)$

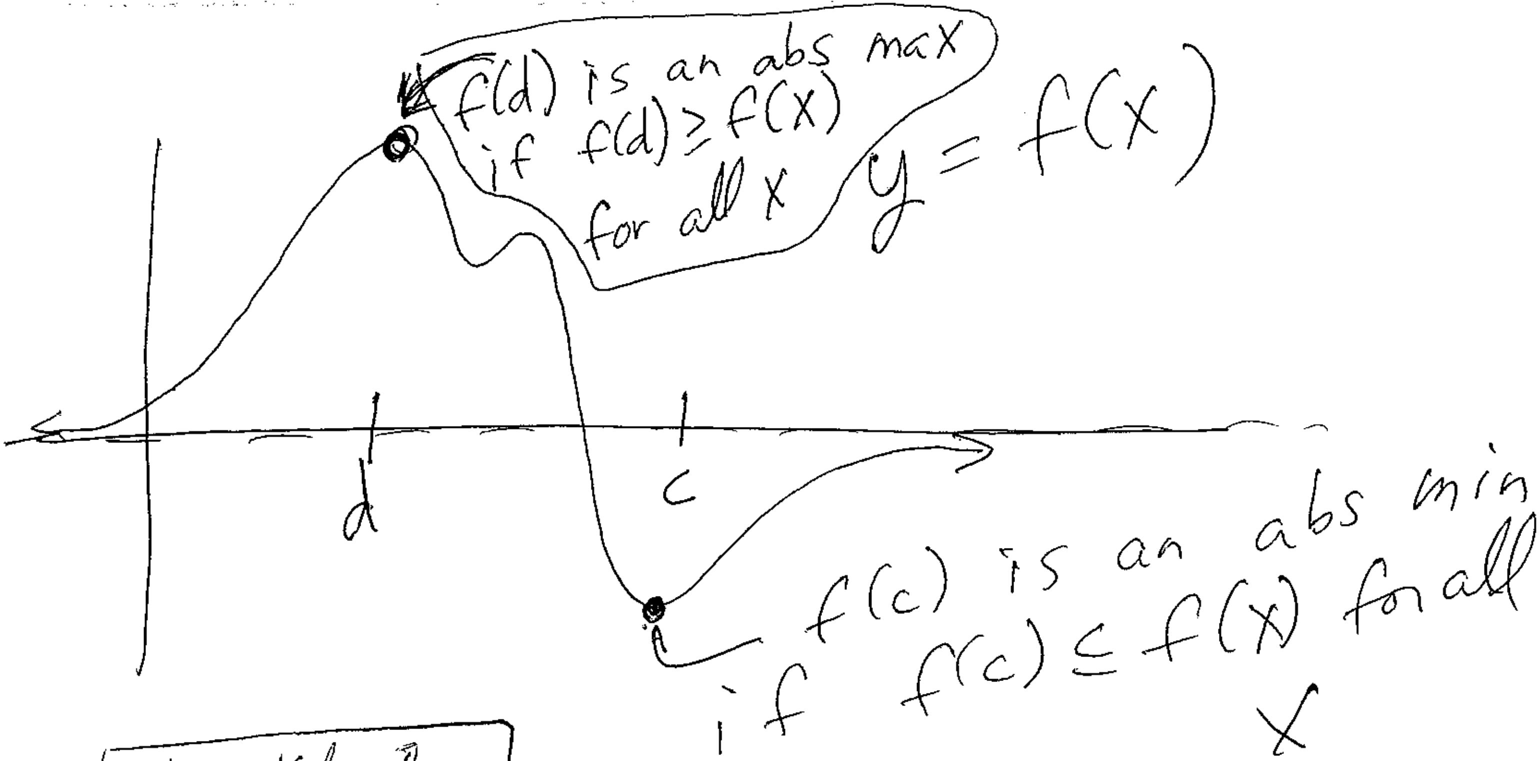
f'' [1.5] 1j.) f is concave down on the intervals $(-1, 0) \cup (0, \infty)$

[1.5] 1k.) Equation(s) of vertical asymptote(s) none

[4] 1l.) Equation(s) of horizontal and/or slant asymptote(s) none

[4.5] 1m.) Graph f





Extreme Value Thm

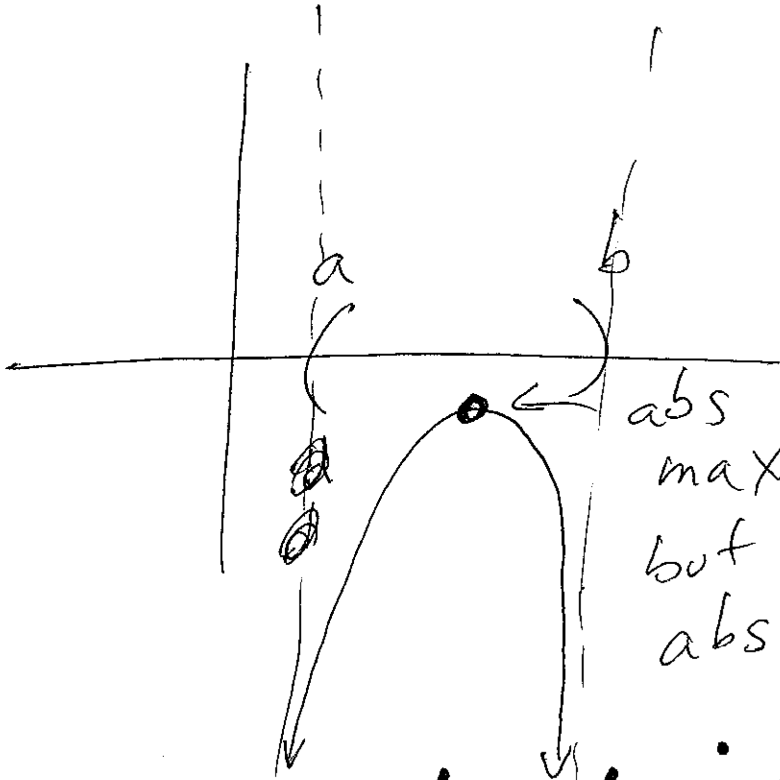
Thm 7 (EVT) If f is cont on $[a, b]$

$\Rightarrow$ there exists $c, d \in [a, b]$ $(a \leq c, d \leq b)$

s.t. $f(c) \leq f(x) \leq f(d)$ for all $x \in [a, b]$

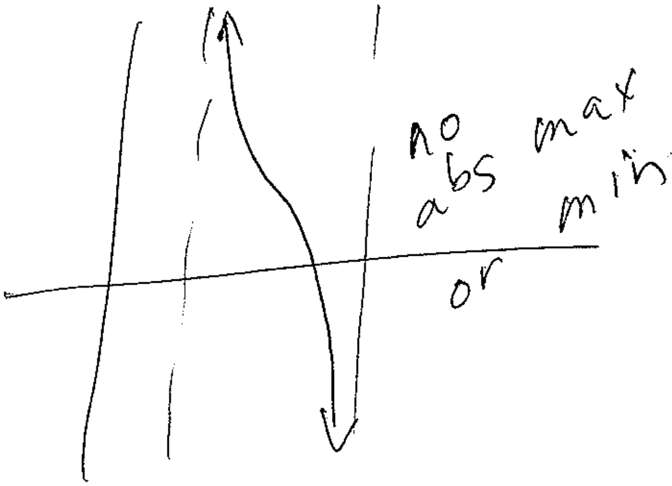
there exists abs min

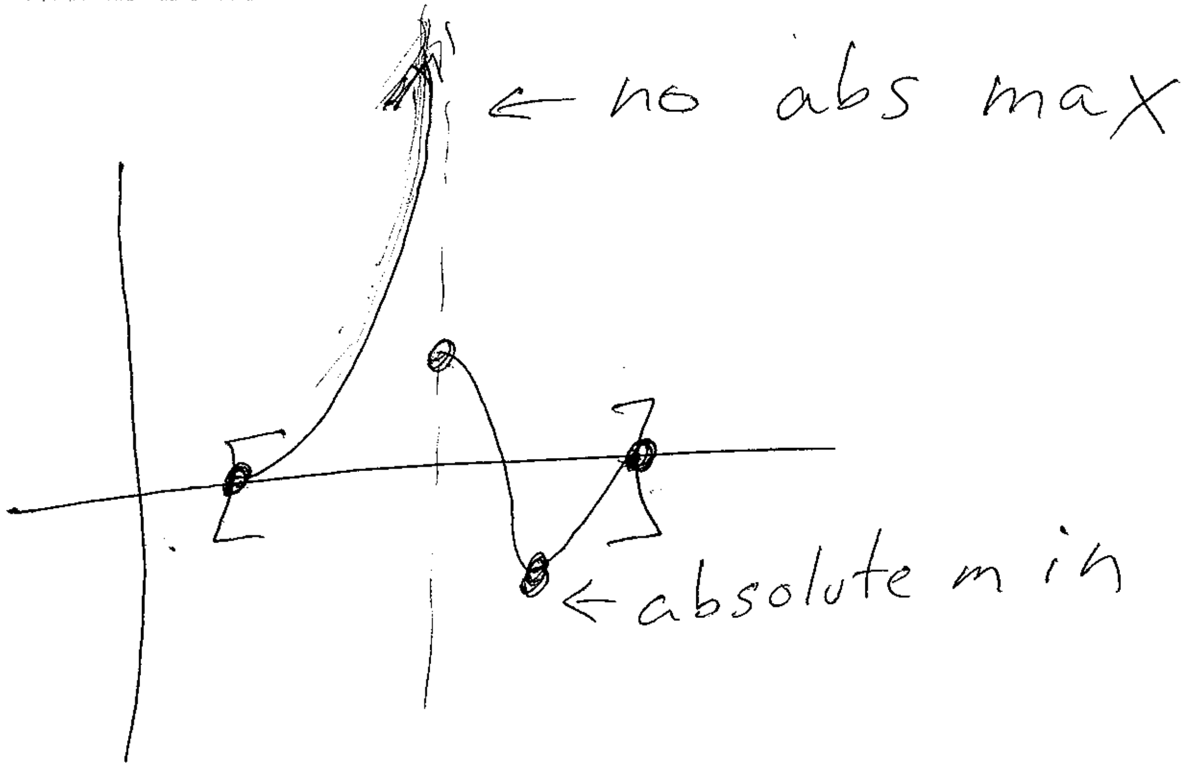
there exists abs max

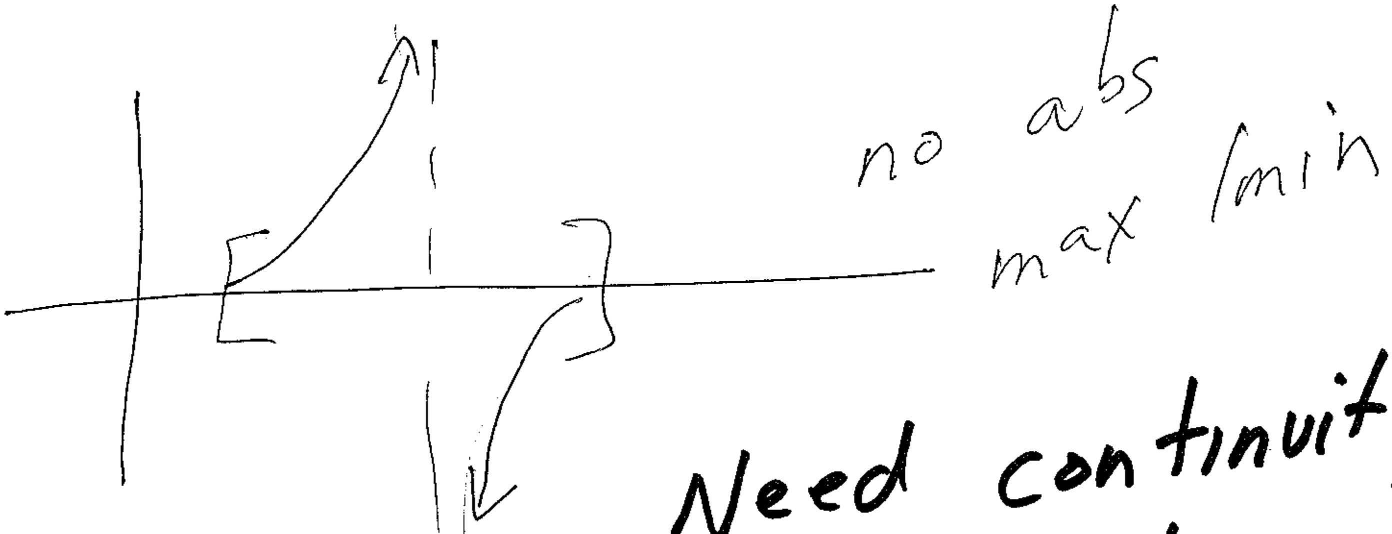


abs max but no abs min

Need closed interval to guarantee existance of abs max/min







Need continuity to guarantee existance of abs max/min

Thm 8: If f cont on $[a,b]$ to find abs max/min

☆ Check critical points & end points

ie abs max/min can only occur at rel max/min or at endpts

Since f is cont $[a,b]$ we know ~~a~~ abs max/min exist so we only need to find them

We know they must occur at rel min/max or at endpts

EX: Find abs max/min

for $f(x) = \sin^2\left(\frac{x}{3}\right) = \left[\sin\left(\frac{x}{3}\right)\right]^2$

on $\left[\frac{3\pi}{4}, \frac{9\pi}{4}\right]$

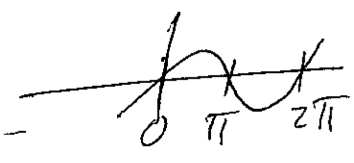
① Find critical points

$$f'(x) = 2\left[\sin\left(\frac{x}{3}\right)\right]\cdot\cos\left(\frac{x}{3}\right)\cdot\left(\frac{1}{3}\right)$$

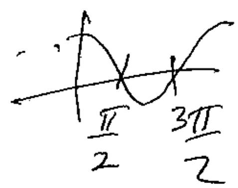
$$= \frac{2}{3}\cdot\sin\left(\frac{x}{3}\right)\cdot\cos\left(\frac{x}{3}\right) = 0, \text{DNE}$$

$$\sin\left(\frac{x}{3}\right) = 0 \Rightarrow \frac{x}{3} = 0, \pi, 2\pi, 3\pi, \dots$$

$$\Rightarrow x = \cancel{0}, \cancel{3\pi}, \cancel{6\pi}, \cancel{9\pi}, \dots$$



$$\cos\left(\frac{x}{3}\right) = 0 \Rightarrow \frac{x}{3} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \dots$$



$$\Rightarrow x = \frac{3\pi}{2}, \cancel{\frac{9\pi}{2}}, \cancel{\dots}$$

Critical pts on $\left[\frac{3\pi}{4}, \frac{9\pi}{4}\right] \Rightarrow x = \frac{3\pi}{2}$

② Check endpts & criticalpts

	x	$y = [\sin(\frac{x}{3})]^2$
critical pts	$\frac{3\pi}{2}$	$[\sin(\frac{\pi}{2})]^2 = 1$ ← abs max
endpts	$\frac{3\pi}{4}$	$[\sin(\frac{\pi}{4})]^2 = \left(\frac{\sqrt{2}}{2}\right)^2 = \frac{1}{2}$
	$\frac{9\pi}{4}$	$[\sin(\frac{3\pi}{4})]^2 = \left(\frac{\sqrt{2}}{2}\right)^2 = \frac{1}{2}$

← abs. min

$y = \sin^2(\frac{x}{3})$ has

abs max $= 1$ at $x = \frac{3\pi}{2}$

abs min $= \frac{1}{2}$ at $x = 3\pi/4, \frac{9\pi}{4}$

in the closed interval

$\left[\frac{3\pi}{4}, \frac{9\pi}{4}\right]$