\[
\frac{f}{g} = f \cdot g^{-1} \Rightarrow -1 \text{ exponent}
\]

\[
\left( \frac{f}{g} \right)' = (f \cdot g^{-1})'
\]

\[
\left( \frac{f(x)}{g(x)} \right)' = \left( f(x) \cdot [g(x)]^{-1} \right)'
\]

\[
= f'(x) \cdot [g(x)]^{-1} + f(x) \cdot (-[g(x)]^{-2} \cdot g'(x))
\]

\[
= \left( f'(x) \cdot [g(x)]^{-1} - f(x) g'(x) \right) \cdot \frac{1}{[g(x)]^2}
\]

\[
= \frac{f'(x) \cdot g(x) - f(x) g'(x)}{[g(x)]^2}
\]
3.1) **Interval defn's**

**open:** $(a, b) = \{ x \mid a < x < b \}$

**closed:** $[a, b] = \{ x \mid a \leq x \leq b \}$

**half open half closed**
FYI: for all $A$ there exists $\exists$
**Theorem 1**: \( f'(x) < 0 \) for all \( x \) in an interval \( I \) implies \( f \) is decreasing over \( I \).

\[ f(x) \text{ is a relative minimum if there exists } (a, b) \text{ st. } c \in (a, b) \text{ s.t. } \forall x \in (a, b), f(x) \geq f(c) \]
$f(c)$ and $f(d)$ are relative extrema

$f(d)$ is a relative maximum if there exist $(e,f)$ such that $d \in (e,f)$ and $f(d) \geq f(x)$ for all $x \in (e,f)$.

$f(c)$ is a relative minimum if there exists $(a,b)$ such that $c \in (a,b)$ [i.e., $a < c < b$] and $f(x) \geq f(c)$ for all $x \in (a,b)$. (or local)
c is a critical point if \( f'(c) = 0 \) or DNE

rel max

rel min's

Note the converse of

SIDENOTE: c should also be in domain of f (but you can ignore that & focus on all points of interest i.e. \( f'(c) = 0 \), DNE
Thm 2: If \( f(c) \) is a relative extrema then \( c \) is a critical point of \( f \).

Note the converse is false. Thm 2 is **not** an if and only if.

Ex: \( f(x) = x^3 \)
\[
f'(x) = 3x^2
\]
\[
3x^2 = 0 \implies x = 0
\]
\( x = 0 \) is a critical point.

But \( f(0) \) is not a relative extrema.

\( f \) is an increasing function even though \( f'(0) = 0 \).
Thm 3: 1st derivative test for finding extrema
Suppose $f$ is cont on $(a, b)$

1) If $f'(x) < 0$ on $\ (a, c)$
   $f'(x) > 0$ on $\ (c, b)$
   \[\Rightarrow\] $f$ has rel min at $c$

\[\begin{array}{cccc}
a & c & b \\
\end{array}\]

2) If $f'(x) > 0$ on $(a, c)$
   $f'(x) < 0$ on $(c, b)$
   \[\Rightarrow\] $f$ has rel max at $c$

\[\begin{array}{cc}
\uparrow & \downarrow \\
\rightarrow & \rightarrow \\
a & c & b \\
\end{array}\]
(3) \( f'(x) > 0 \) on \( (a, c) \) \& \( (c, b) \)  
\[ \Rightarrow \] no rel ext at \( c \)

\[ f'(x) < 0 \] on \( (a, c) \) \& \( (c, b) \)  
\[ \Rightarrow \] no rel ext at \( c \)

\[ f(x) = x^2 + x - 6 \]

\[ f'(x) = 2x + 1 \]

\[ 2x + 1 = f'(x) \]

\[ \text{rel min} \]

\[ 2x + 1 = 0 \]

\[ -\frac{1}{2} \]