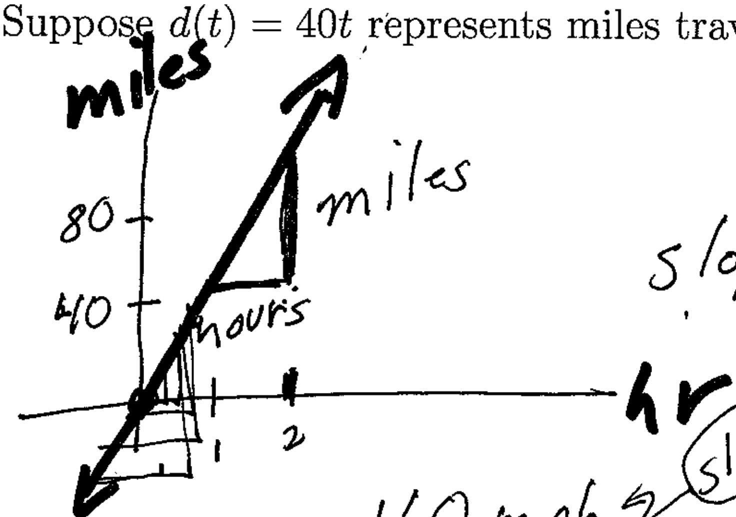
2.3 average rate of change

Suppose $d(t) = 40t$ represents miles traveled after t hours.



$t=0$ & $t=2$

$$\text{slope} = \frac{d(2)-d(0)}{2-0} = \frac{80}{2} = 40$$

$t=0$ & $t=1$

$\frac{40}{1} = 40$ mph

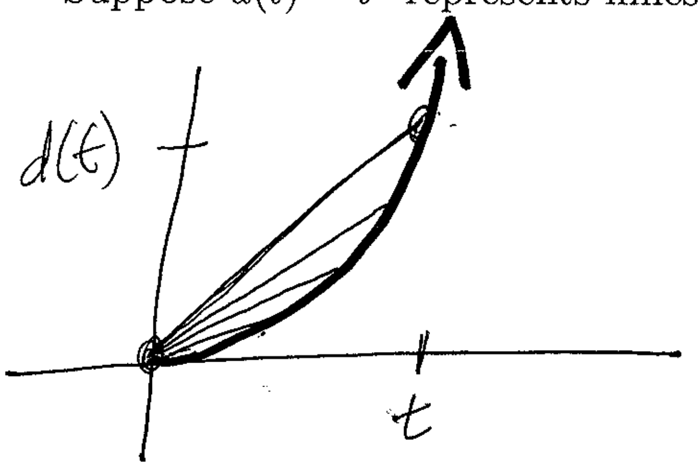
Average velocity is 40 mph slope

$t=0$ & $t=\frac{1}{2}$

$\frac{20-0}{\frac{1}{2}-0} = 40$

2.6 Instantaneous velocity at $t = t_0$ is 40mph

Suppose $d(t) = t^2$ represents miles traveled after t hours.



average velocity between t_0 & t

$$\text{slope} = \frac{d(t_0)-d(t)}{t_0-t} = \frac{\text{change in distance}}{\text{change in time}} = \frac{\Delta d}{\Delta t}$$

t	change in time btwn $t_0 = 0$ and t	change in distance btwn $t_0 = 0$ and t	average velocity btwn $t_0 = 0$ and t
2	$2-0$	2^2-0^2	$\frac{2^2-0^2}{2-0} = 2$ mph
1	$1-0$	1^2-0^2	$\frac{1^2-0^2}{1-0} = 1$ mph
.5	$.5-0$	$(.5)^2-0^2$	$\frac{(.5)^2-0^2}{.5-0} = .5$ mph
.1	$.1-0$	$(.1)^2-0^2$	$\frac{(.1)^2-0^2}{.1-0} = .1$ mph
.01	$.01-0$	$(.01)^2-0^2$	$\frac{(.01)^2-0^2}{.01-0} = .01$ mph
	$t-t_0$	$d(t)-d(t_0)$	

2.6 Instantaneous velocity at $t_0 = 0$ is 0mph

Average velocity btwen

$t = 0$ & $t = 2$

$$\frac{\text{change in distance}}{\text{change in hrs}} = \frac{\Delta d}{\Delta t} = \frac{d(2) - d(0)}{2 - 0}$$

$$= \frac{80 - 0}{2 - 0} = \frac{80}{2} = 40 \text{ mph}$$

$t = 0$ & $t = 1$

$$= \frac{40}{1} = 40 \text{ mph}$$

$t = 3, \quad t = 20$

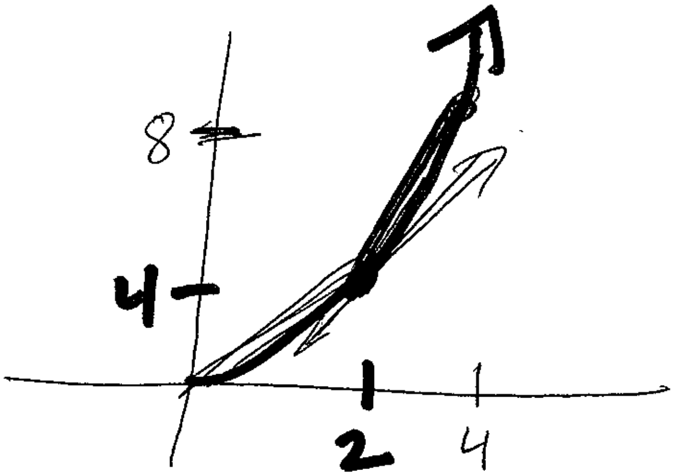
$$= \frac{d(20) - d(3)}{20 - 3} = \frac{(40)(20) - 40(3)}{20 - 3}$$

$$= \frac{40[20 - 3]}{20 - 3} = 40 \text{ mph}$$

Instaneous velocity

$$= \lim_{t \to t_0} \frac{d(t) - d(t_0)}{t - t_0}$$ 3.6

Suppose $d(t) = t^2$ represents miles traveled after t hours.



$$\frac{4^2-2^2}{4-2} = \frac{2^2-4^2}{2-4}$$

$= \text{slope}$

t	change in time btwn $t_0 = 2$ and t	change in distance btwn $t_0 = 2$ and t	average velocity btwn $t_0 = 2$ and t
4	$4-2$	4^2-2^2	$\frac{4^2-2^2}{4-2} = 6$ mph
3	$3-2$	3^2-2^2	$\frac{3^2-2^2}{3-2} = 5$ mph
2.5	$2.5-2$	$(2.5)^2-2^2$	$\frac{(2.5)^2-2^2}{2.5-2} = 4.5$ mph
2.1	$2.1-2$	$(2.1)^2-2^2$	$\frac{(2.1)^2-2^2}{2.1-2} = 4.1$ mph
1.9	$1.9-2$	$(1.9)^2-2^2$	$\frac{(1.9)^2-2^2}{1.9-2} = 3.9$ mph
1.5	$1.5-2$	$(1.5)^2-2^2$	$\frac{(1.5)^2-2^2}{1.5-2} = 3.5$ mph
1	$1-2$	1^2-2^2	$\frac{1^2-2^2}{1-2} = 3$ mph

Instantaneous velocity at $t_0 = 2$ is 4 mph

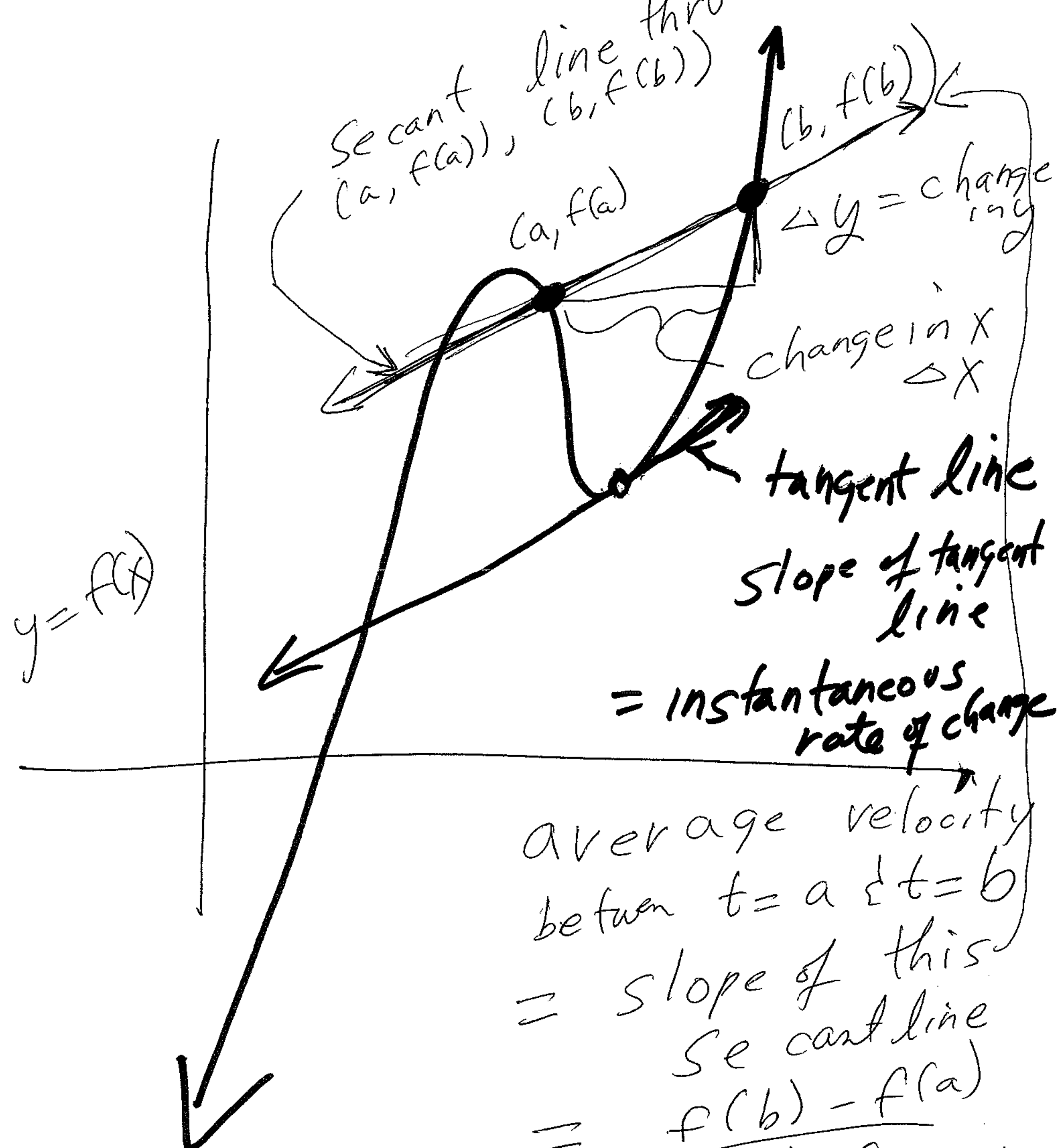
SLOPE OF SECANT LINE = AVERAGE VELOCITY

SLOPE OF TANGENT LINE = INSTANTANEOUS VELOCITY

in general, SLOPE = RATE OF CHANGE

2.3 SLOPE OF SECANT LINE = AVERAGE RATE OF CHANGE

2.6 SLOPE OF TANGENT LINE = INSTANTANEOUS RATE OF CHANGE



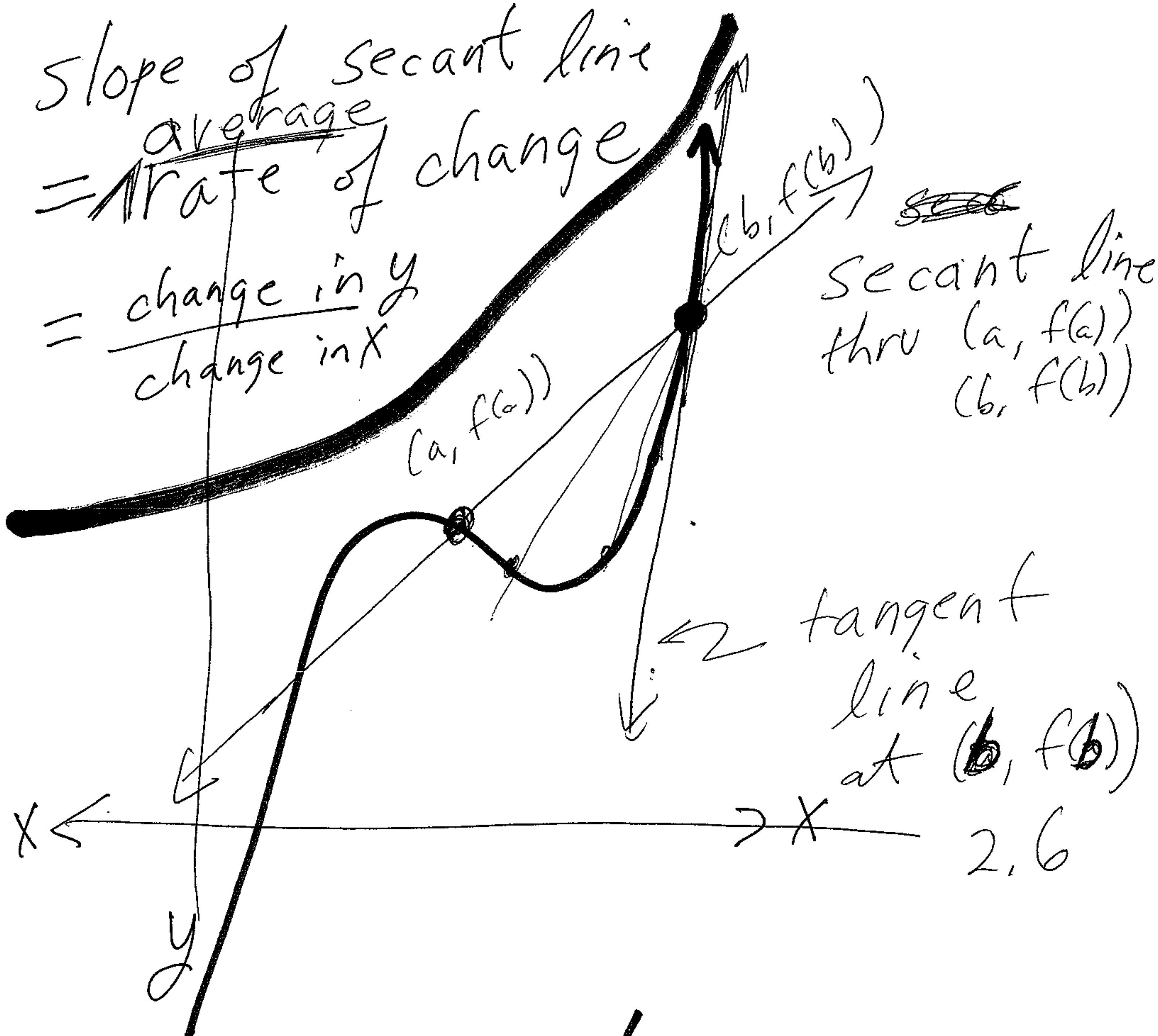
slope of tangent line
= instantaneous rate of change

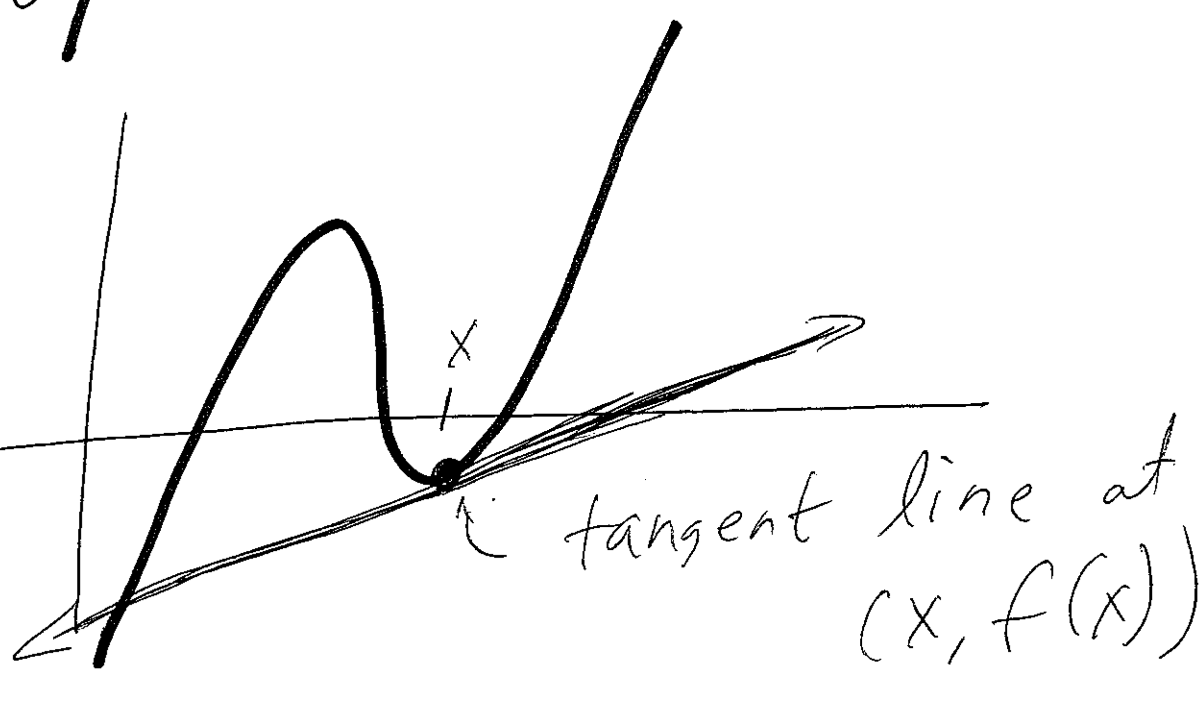
average velocity
betwen $t = a$ & $t = b$
= slope of this secant line
$$= \frac{f(b) - f(a)}{b - a}$$
= average rate of change

slope of secant line

= average rate of change

$= \dfrac{\text{change in } y}{\text{change in } X}$





slope of secant line
[ie line thru $(x, f(x))$
$(x_0, f(x_0))$]

$=$ average rate of change btwn $(x, f(x))$, $(x_0, f(x_0))$

$$= \frac{f(x) - f(x_0)}{x - x_0}$$

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average rate of change btwn $(x_0+h, f(x_0+h))$ & $(x_0, f(x_0))$

$$= \frac{f(x_0+h) - f(x_0)}{x_0+h - x_0}$$

$$= \frac{f(x_0+h) - f(x_0)}{h}$$

average rate of change btwn $(x+h, f(x+h))$ & $(x, f(x))$

= slope of secant line thru these points

$$= \frac{f(x+h) - f(x)}{x+h-x}$$

$$= \boxed{\frac{f(x+h) - f(x)}{h}}$$

$f(x) = x^2 - 2x$

Find average rate of change btwn $(x+h, f(x+h))$ & $(x, f(x))$

$= \text{slope}$

$$= \frac{f(x+h) - f(x)}{x+h-x}$$

$$= \frac{f(x+h) - f(x)}{h}$$

$$= \frac{(x+h)^2 - 2(x+h) - [x^2 - 2x]}{h}$$

$$= \frac{\cancel{x^2} + 2xh + h^2 - \cancel{2x} - 2h - \cancel{x^2} + \cancel{2x}}{h}$$

$$= \frac{2xh + h^2 - 2h}{h}$$

$$= \frac{\cancel{h}(2x + h - 2)}{\cancel{h}}$$

$$= \boxed{2x + h - 2}$$