2.3 | average rate of change

Suppose $d(t) = 40t$ represents miles traveled after $t$ hours.

\[
\begin{align*}
\text{average velocity is } & 40 \text{ mph} \\
\text{slope: } & \frac{d(2) - d(0)}{2 - 0} = \frac{80}{2} = 40 \\
\text{slope: } & \frac{d(1) - d(0)}{1 - 0} = \frac{40}{1} = 40 \text{ mph} \\
\text{slope: } & \frac{d(\frac{1}{2}) - d(0)}{\frac{1}{2} - 0} = 40 \\
\end{align*}
\]

2.6 | Instantaneous velocity at $t = t_0$ is $40 \text{ mph}$

Suppose $d(t) = t^2$ represents miles traveled after $t$ hours.

\[
\begin{align*}
\text{average velocity between } & t_0 \leq t \\
\text{slope: } & \frac{d(t_0) - d(t)}{t_0 - t} \\
\text{change in distance } & \frac{\Delta d}{\Delta t} \\
\end{align*}
\]

<table>
<thead>
<tr>
<th>$t$</th>
<th>$2 - 0$</th>
<th>$1 - 0$</th>
<th>$.5 - 0$</th>
<th>$.1 - 0$</th>
<th>$.01 - 0$</th>
<th>$\frac{2^2 - 0^2}{2 - 0} = 2 \text{ mph}$</th>
<th>$\frac{1^2 - 0^2}{1 - 0} = 1 \text{ mph}$</th>
<th>$\frac{(.5)^2 - 0^2}{.5 - 0} = .5 \text{ mph}$</th>
<th>$\frac{(.1)^2 - 0^2}{.1 - 0} = .1 \text{ mph}$</th>
<th>$\frac{(.01)^2 - 0^2}{.01 - 0} = .01 \text{ mph}$</th>
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</tbody>
</table>

2.6 | Instantaneous velocity at $t_0 = 0$ is $0 \text{ mph}$
Average velocity between 
\( t = 0 \) and \( t = 2 \)

\[
\text{Change in distance} \over \text{change in hrs} = \frac{\Delta d}{\Delta t} = \frac{d(2) - d(0)}{2 - 0}
\]

\[
= \frac{80 - 0}{2 - 0} = \frac{80}{2} = 40 \text{ mph}
\]

\( t = 0 \) and \( t = 1 \)

\[
= \frac{40}{1} = 40 \text{ mph}
\]

\( t = 3 \) and \( t = 20 \)

\[
= \frac{d(20) - d(3)}{20 - 3} = \frac{(40)(20) - 40(3)}{20 - 3}
\]

\[
= \frac{40(20 - 3)}{20 - 3} = 40 \text{ mph}
\]
Instantaneous velocity:

\[
\lim_{t \to t_0} \frac{d(t) - d(t_0)}{t - t_0} = 2 \text{ m/s}
\]
Suppose \( d(t) = t^2 \) represents miles traveled after \( t \) hours.

\[
\frac{y^2 - 2^2}{y - 2} = \frac{2^2 - 4^2}{2 - 4}
\]

\[= \frac{5}{2}\]

\[
t \quad \text{change in time} \quad \text{between} \; t_0 = 2 \; \text{and} \; t \\
4 \quad 4 - 2 \quad 4^2 - 2^2 \quad \frac{4^2 - 2^2}{4 - 2} = 6 \text{ mph}
3 \quad 3 - 2 \quad 3^2 - 2^2 \quad \frac{3^2 - 2^2}{3 - 2} = 5 \text{ mph}
2.5 \quad 2.5 - 2 \quad (2.5)^2 - 2^2 \quad \frac{(2.5)^2 - 2^2}{2.5 - 2} = 4.5 \text{ mph}
2.1 \quad 2.1 - 2 \quad (2.1)^2 - 2^2 \quad \frac{(2.1)^2 - 2^2}{2.1 - 2} = 4.1 \text{ mph}
1.9 \quad 1.9 - 2 \quad (1.9)^2 - 2^2 \quad \frac{(1.9)^2 - 2^2}{1.9 - 2} = 3.9 \text{ mph}
1.5 \quad 1.5 - 2 \quad (1.5)^2 - 2^2 \quad \frac{(1.5)^2 - 2^2}{1.5 - 2} = 3.5 \text{ mph}
1 \quad 1 - 2 \quad 1^2 - 2^2
\]

Instantaneous velocity at \( t_0 = 2 \) is ______ mph.

SLOPE OF SECANT LINE = AVERAGE VELOCITY

SLOPE OF TANGENT LINE = INSTANTANEOUS VELOCITY

in general, SLOPE = RATE OF CHANGE

SLOPE OF SECANT LINE = AVERAGE RATE OF CHANGE

SLOPE OF TANGENT LINE = INSTANTANEOUS RATE OF CHANGE
\[ y = f(x) \]

Secant line thru \((a, f(a)), (b, f(b))\)

\[ \Delta y = \text{change in } y \]
\[ \Delta x = \text{change in } x \]

Tangent line
Slope of tangent line

= instantaneous rate of change

Average velocity before \( t = a \) \& \( t = b \)

= slope of this Secant line

= \( \frac{f(b) - f(a)}{b - a} \)

= average rate of change
Slope of secant line = average rate of change
= \frac{\text{change in } y}{\text{change in } x}

Secant line thru \((a, f(a))\)
\((b, f(b))\)

Tangent line at \((b, f(b))\)

Tangent line at \((x, f(x))\)
Slope of secant line
\[ \text{[line thru } (x, f(x)), (x_0, f(x_0)) \text{]} \]

= average rate of change between \((x, f(x)), (x_0, f(x_0))\)

= \[ \frac{f(x) - f(x_0)}{x - x_0} \]

Average rate of change between \((x_0 + h, f(x_0 + h)), (x_0, f(x_0))\)

= \[ \frac{f(x_0 + h) - f(x_0)}{x_0 + h - x_0} \]

= \[ \frac{f(x_0 + h) - f(x_0)}{h} \]
average rate of change between
\((x + h, f(x + h))\) and \((x, f(x))\)

= slope of secant line
thru these points

= \[ \frac{f(x + h) - f(x)}{x + h - x} \]

= \[ \frac{f(x + h) - f(x)}{h} \]
\[ f(x) = x^2 - 2x \]

Find average rate of change
between \((x+h, f(x+h))\) and \((x, f(x))\).

\[
\text{Slope} = \frac{f(x+h) - f(x)}{h}
\]

\[
= \frac{(x+h)^2 - 2(x+h) - (x^2 - 2x)}{h}
\]

\[
= \frac{x^2 + 2hx + h^2 - 2x - 2h - x^2 + 2x}{h}
\]

\[
= \frac{2hx + h^2 - 2h}{h}
\]
\[
\begin{align*}
\lim_{h \to 0} \frac{2xh + h^2 - 2h}{h} &= \frac{h(2x + h - 2)}{h} \\
&= 2x + h - 2
\end{align*}
\]