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Today OH 3:30 – 5:20 in MLH 110

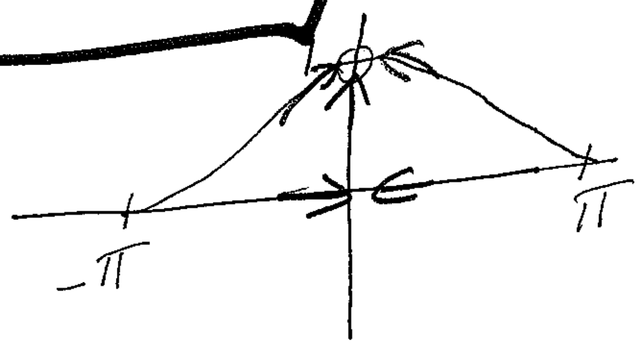
↑ notes + HW

Find limits

2 - methods

① Graphically

EX: $\boxed{\lim_{x \to 0} \frac{\sin x}{x} = 1}$



② Algebraically

too messy for $\lim_{x \to 0} \frac{\sin x}{x}$

See next page(s) for other examples.

$\lim_{x \to a} f(x)$

$\frac{\text{"}\#\text{"}}{\#}$, $\frac{\text{"}\#\text{"}}{0}$ $\neq 0$, $\frac{\text{"}0\text{"}}{0}$

Both $\neq 0$ ✓

DNE

simplify

Function f is nice at $x=a$ if I can estimate $f(a \pm 0.001)$ with $f(a)$

Ex $f(x) = x^2$

$f(2.001) = (2.001)^2 \sim 2^2 = f(2)$

$f(2.001) \sim f(2)$

f is nice at 2

$\boxed{\text{nice} = \text{continuous}}$

Hence can evaluate $\lim_{x \to a} f(x)$ by plugging in a **if**

$lim_{x\to 3}\frac{x^2-1}{x+3} = \frac{3^2-1}{3+3} = \frac{9-1}{6} = \frac{8}{6} = \boxed{\frac{4}{3}}$

$lim_{x\to 3}\ \frac{x^2-1}{x-3} = DNE$

$\text{"}\frac{3^2-1}{0}\text{"}$ $\neq 0$ $\text{"}\frac{\text{nonzero}}{\text{zero}}\text{"}$

$lim_{x\to 3}\ \frac{(x^2-1)\cancel{(x-3)}}{\cancel{x-3}} = \lim_{x\to 3} x^2-1 = \boxed{8}$

$\text{"}\frac{0}{0}\text{"} \Rightarrow$ simplify

$lim_{x\to 3}\ \frac{x-3}{x^2-1} = 0$

$\text{"}\frac{0}{8}\text{"}$

$lim_{x\to 3}\ \frac{(x-4)^2}{x^5(x-8)^9(x-3)^3} = DNE$

$\text{"}\frac{\#}{0}\text{"}$ $\neq 0$

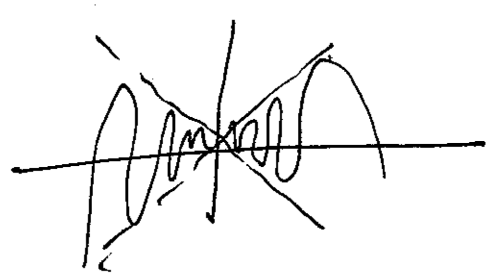
$lim_{x\to 3}\ \frac{(x-4)^2\cancel{(x-3)}}{x^5(x-8)^9(x-3)^{\cancel{3}\,2}} = \lim_{x\to 3}\frac{(x-4)^2}{x^5(x-8)^9(x-3)^2} = DNE$

$\text{"}0\text{"}/0$

$\boxed{\text{Challenge}}$ example: $g(x) = xsin\frac{1}{x}$

not on exam

$-|x| \le xsin\frac{1}{x} \le |x|$



$lim_{x\to 0}\big(-|x|\big) = 0,\ lim_{x\to 0}\big(|x|\big) = 0.$

Hence , $lim_{x\to 0}\big(xsin\frac{1}{x}\big) = \underline{0}$

h is a variable but everything else = constants

x is constant

Standard example:

Suppose $f(x) = \sqrt{x}$ Find $lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ where $x > 0$

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{x+h} - \sqrt{x}}{h}$$

trick

"$\frac{0}{0}$" simplify

$$= \lim_{h \to 0} \left(\frac{\sqrt{x+h} - \sqrt{x}}{h}\right)\left(\frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}\right)$$

$$= \lim_{h \to 0} \frac{(\cancel{x} + h) - \cancel{x}}{h(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h \to 0} \frac{\cancel{h} \cdot 1}{\cancel{h}(\sqrt{x+h} + \sqrt{x})}$$

$$= \lim_{h \to 0} \frac{1}{\sqrt{x+h} + \sqrt{x}} = \frac{1}{\sqrt{x} + \sqrt{x}} = \boxed{\frac{1}{2\sqrt{x}}}$$

$$f(x) = \sqrt{x}$$

$$f(\text{blah}) = \sqrt{\text{blah}}$$

$$f(\square) = \sqrt{\square}$$

$$f(\boxed{x+h}) = \sqrt{\boxed{x+h}}$$

$$f(x+h) = \sqrt{x+h}$$

query for **x**
replace w/ **x+h**

Suppose $c \in \mathcal{R}$ and suppose $\lim_{x\to a} f(x)$ and $\lim_{x\to a} g(x)$ exist. Then

$$\lim_{x\to a}[f(x) + g(x)] = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$$

$$\lim_{x\to a}[cf(x)] = c \lim_{x\to a} f(x)$$

$$\lim_{x\to a}[f(x)g(x)] = \lim_{x\to a} f(x) \lim_{x\to a} g(x)$$

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)} \text{ if } \lim_{x\to a} g(x) \neq 0$$

Defn: f is continuous at a if $\lim_{x\to a} f(x) = f(a)$
(i.e., if $\lim_{x\to a} f(x) = f(\lim_{x\to a} x)$)

plug in a to find limit

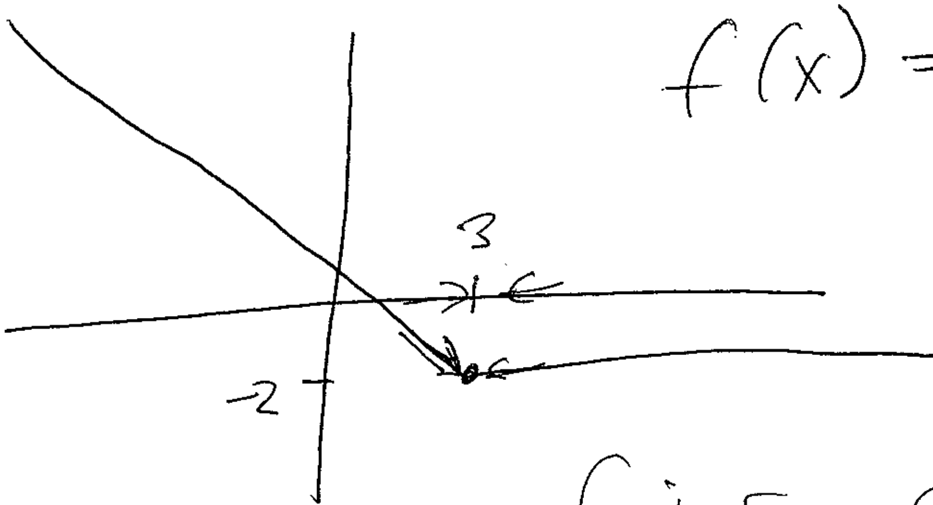
In other words, f is continuous at a if
1.) $f(a)$ exists,
2.) $\lim_{x\to a} f(x)$ exists, and
3.) $\lim_{x\to a} f(x) = f(a)$

Defn: f is continuous is f is continuous at a for every a in the domain of f.

Examples:

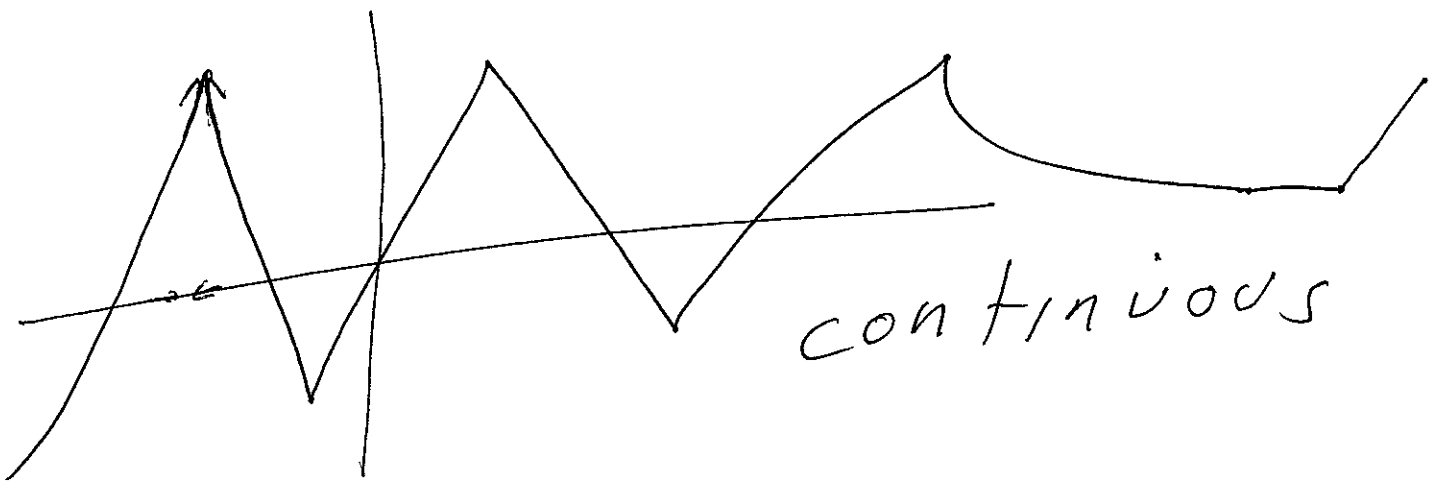
$$f(x) = 2x^3 + \sqrt{\cos(e^x)} + \frac{\sqrt[3]{3}}{\log(x^2+1)}$$

Ex: Polynomial, rational, root, trigonometric, inverse trigonometric, exponential, logarithmic functions are continuous functions.

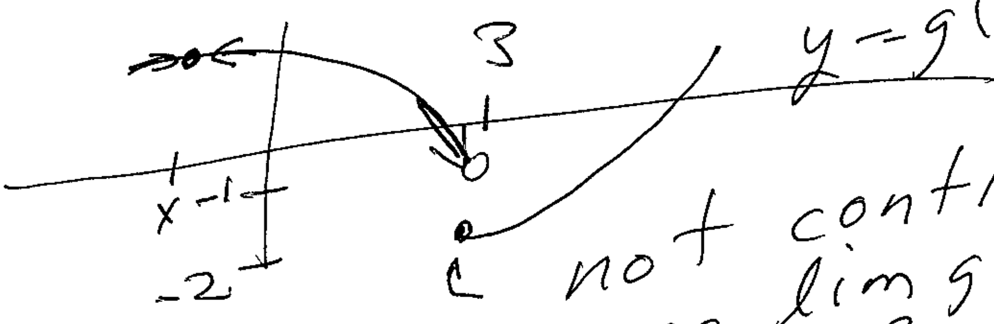


$$f(x) = \begin{cases} -x+1, & x>3 \\ -2, & x\le 3 \end{cases}$$

f is continuous

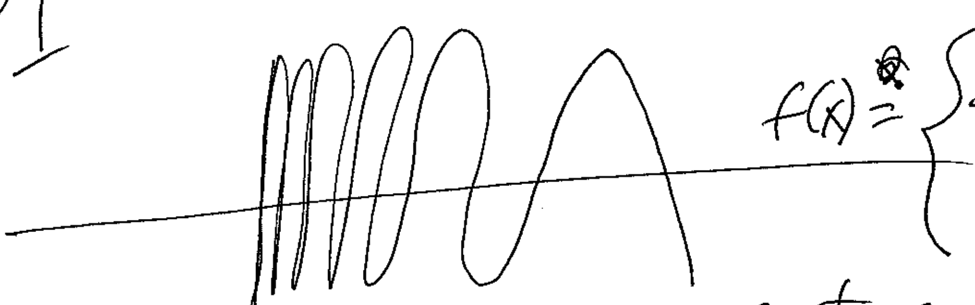


continuous



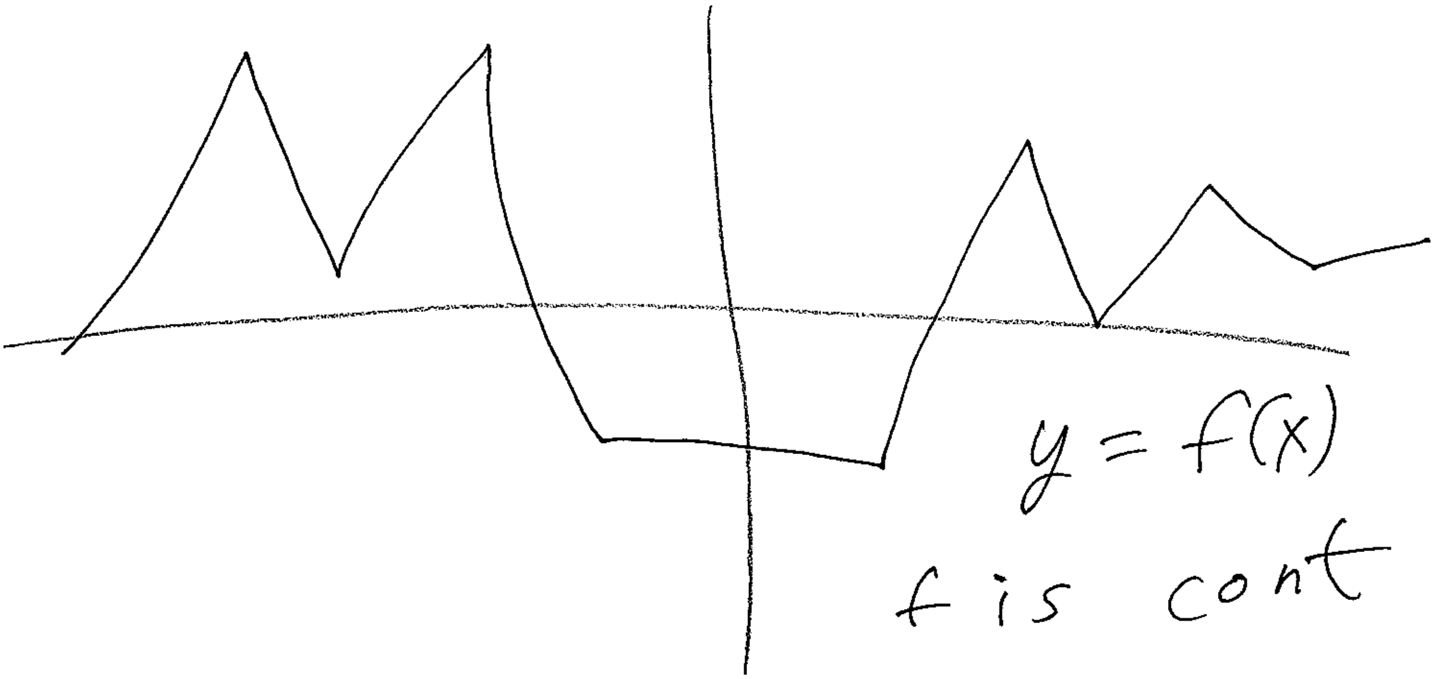
not continuous since $\lim_{x\to 3} g(x) = DNE$

FYI



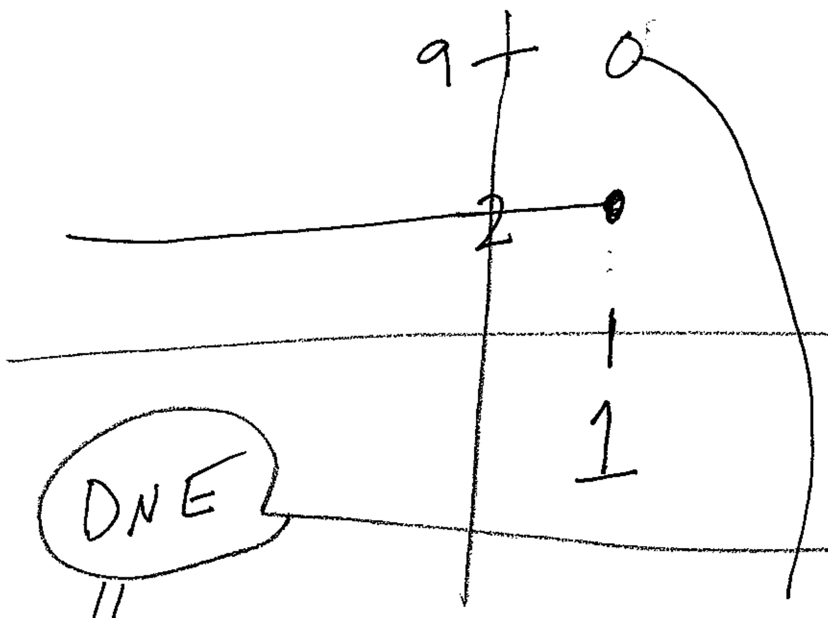
$$f(x) = \begin{cases} \sin\left(\frac{1}{x}\right), & x\neq 0 \\ 0, & x=0 \end{cases}$$

not cont at 0



$y = f(x)$

f is cont



$y = g(x) = \begin{cases} 2 & x \le 1 \\ x^2 - 1 & x > 1 \end{cases}$

$y = g(x)$

not cont at 1

ONE

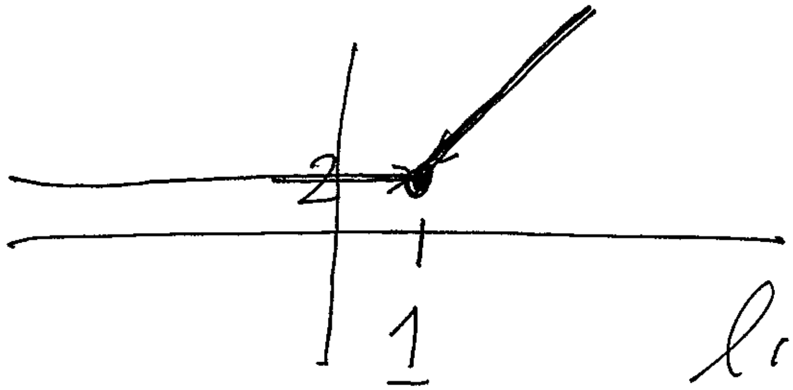
$\lim_{x \to 1} g(x) \neq g(1)$

so its not cont

But g is cont at a for all $a \neq 1$

Ex: $\lim_{x \to 2} g(x) = 2^2 - 1 = 4 - 1 = 3$

$$h(x) = \begin{cases} 2 & x \le 1 \\ x+1 & x > 1 \end{cases}$$

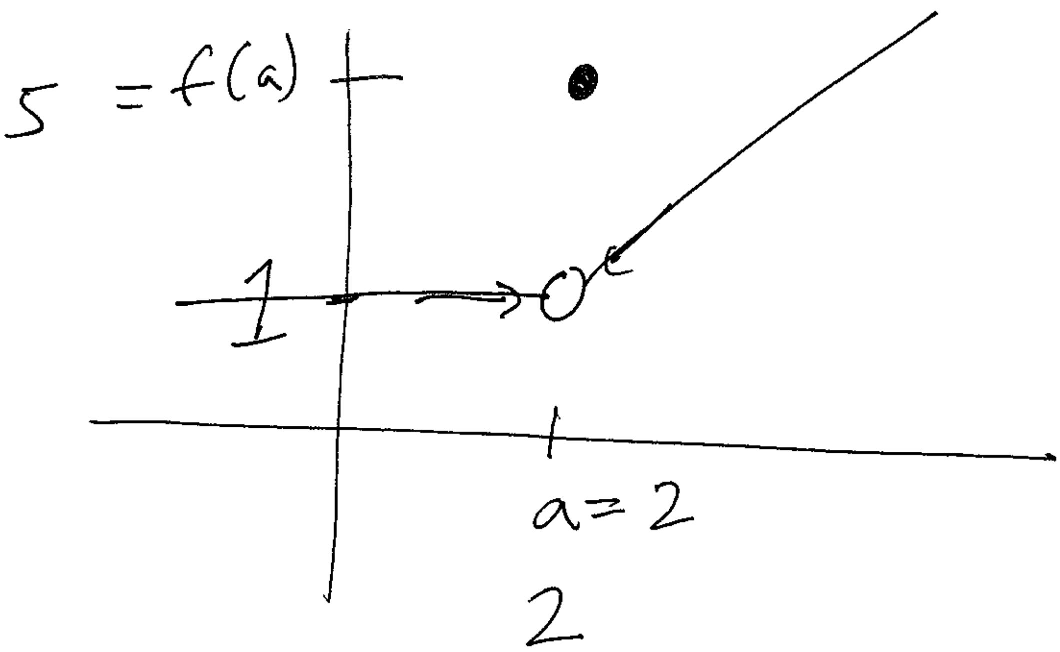


h is cont

$$\lim_{x \to 1} h(x) = 2$$

$$\lim_{x \to a} h(x) = h(a)$$

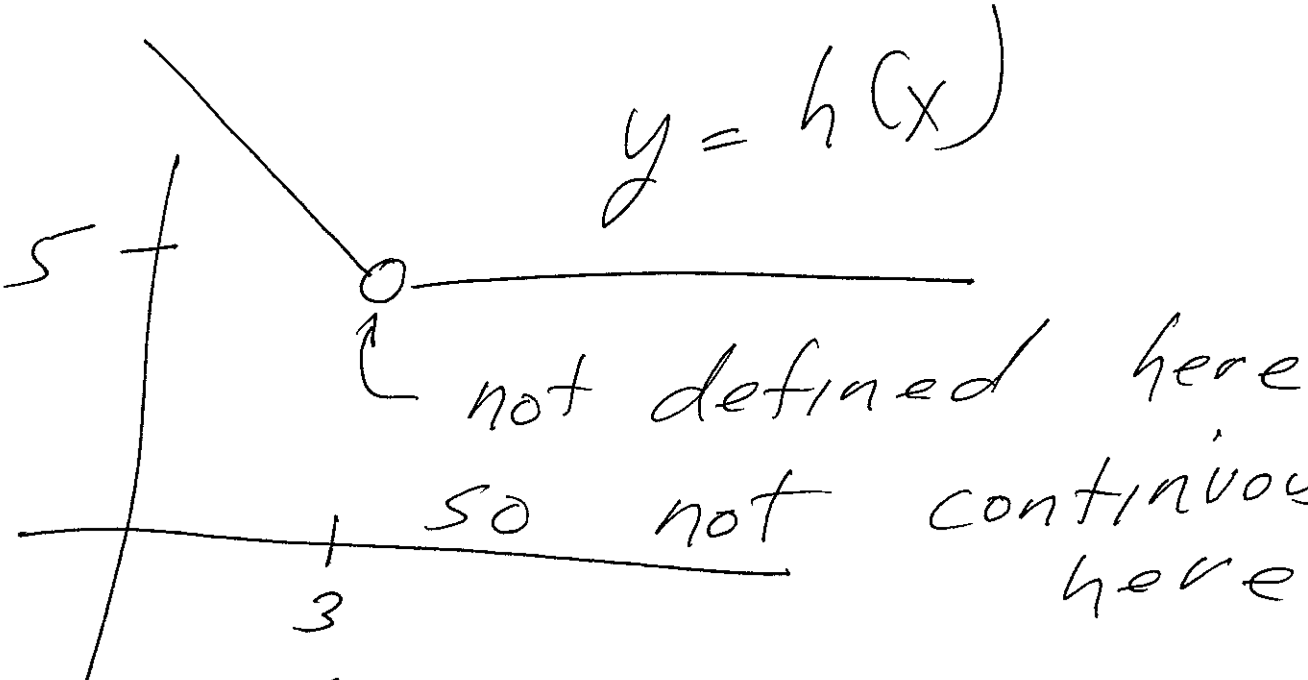
since h is cont



$$\lim_{x \to 2} f(x) = 1 \neq f(1) = 5$$

f is not cont
at $X = 1$

f is not cont.



$\lim_{x \to 3} h(x)$ exist

$$\lim_{x \to 3} h(x) = 5$$

but h is not cont at 3

If f, g continuous at a, $c \in \mathcal{R}$, then $f+g$, fg, cf, f/g (if $g(a) \neq 0$) are continuous at a.

If g continuous at a and f continuous at $g(a)$, then $f \circ g$ continuous at a.

Ex: $lim_{x \to 0} \frac{x^2 - e^{x^3}}{cos(x)} = \frac{0^2 - e^{0^3}}{\cos(0)} = \frac{0-1}{1} = -1$

Ex: $lim_{x \to 9} e^{\sqrt{x}} - 2\sqrt{x} + 4 = e^{\sqrt{9}} - 2\sqrt{9} + 4$

$= e^3 - 6 + 4 = \boxed{e^3 - 2}$

Ex: $lim_{x \to 0} cos(sin(x)) = \cos(\sin(0))$

$= \cos(0) = \underline{1}$

Ex: $lim_{x \to 0} cos(\frac{sin(x)}{x}) = \boxed{\cos(1)}$

$\parallel$

$\cos\left(\lim_{x \to 0}\left(\frac{\sin x}{x}\right)\right)$ ← skip this step.

Ex: $lim_{h \to 0} (h)\underbrace{tan(x)}_{\text{constant}}csc(h) = \tan(x)\left[\lim_{h \to 0} h \csc(h)\right]$

h is the variable

$= \tan(x)\left[\lim_{h \to 0} \frac{h}{\sin h}\right]$

$= \tan(x)[1] = \boxed{\tan x}$

$$\lim \frac{h}{\sin h} = \lim \frac{1}{\frac{\sin h}{h}}$$

$$\frac{\sin h}{h} \to 1$$

$$\frac{h}{\sin h} = \underbrace{\left(\frac{\sin h}{h}\right)}^{-1}$$

$$\frac{h}{\sin h} \to 1^{-1} = \underline{1} \qquad \left(\frac{\sin h}{h}\right)^{-1} \to (1)^{-1}$$