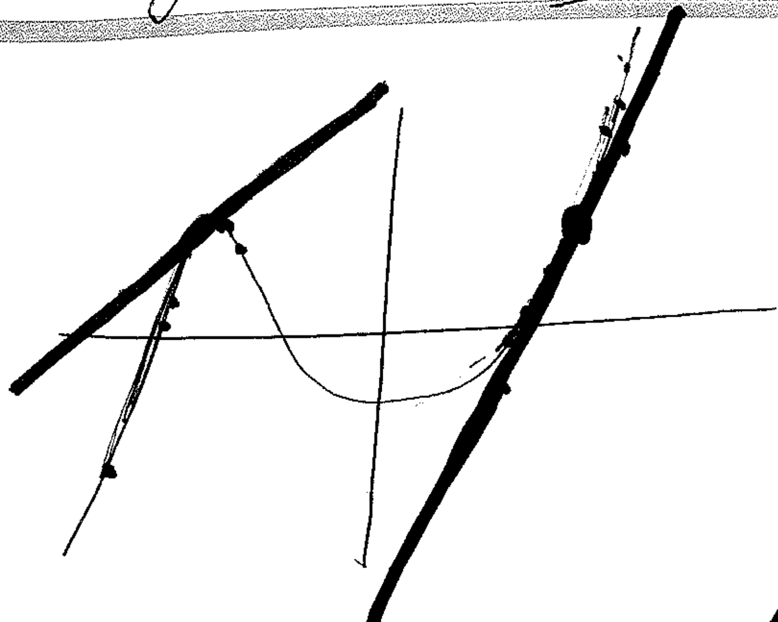
Given $y = f(x)$
the tangent line
to $y = f(x)$ at $x = a$
is a good approx
to $y = f(x)$ near a



$f(x) \approx mx+b$
for x close
to a

3.6 : Solve $f(x) = 0$

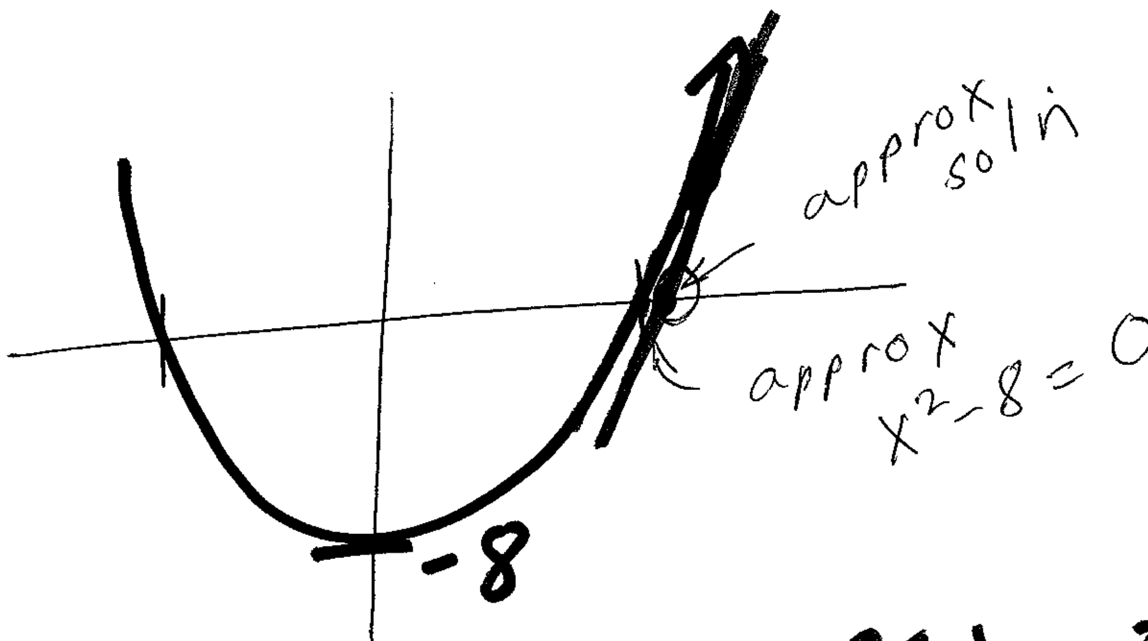
Note tangent at $x=a$ is
good approx to $y=f(x)$ near $x=a$

$y = mx + b$ ← tangent line

Solve: $mx+b=0$ to approx $f(x)=0$

Solve $x^2 - 8 = 0$

$f(x) = x^2 - 8 = 0$



Find tangent line to $f(x) = x^2 - 8$ at $x = 3$

slope: $f'(x) = 2x$

$f'(3) = 6$

point on line $(3, f(3))$

$(3, 1)$

$$\frac{y-1}{x-3} = 6$$

$$y - 1^{+1} = 6(x-3) = 6x - 18^{+1}$$

$$y = 6x - 17$$ tangent line

$$\boxed{\text{Near } x = 3 \quad f(x) = x^2 - 8 \sim 6x - 17}$$

Solve $x^2 - 8 = 0$

Approx $6x - 17 = 0$

$$\Rightarrow \boxed{x = \frac{17}{6}}$$

first approx using Newton's method

When to use log-log paper:

Suppose you suspect your data points satisfy polynomial growth of the form $y = At^m$ for some constants A and m.

$y = At^m \qquad \boxed{10^b t^m = y}$
$log(y) = log(At^m)$
$(log(y)) = log(A) + m(log(t))$ Let $z = log(y)$ and $x = log(t)$. Then
$z = log(A) + mx$.

$z = mx + log(A)$. That is we have the equation of a line where slope $= m$ and z-intercept $= log(A)$.

If $z = mx + b$, then $log(A) = b$. Hence $A = 10^{log(A)} = 10^b$.

Hence to determine the constants A and m in $y = At^m$, graph (t, y) on log-log paper (note this is the same as taking $z = log(y)$ and $x = log(t)$), and determine equation of best fit line, $z = mx + b$. Then $y = 10^b t^m$.

However if the data points do not satisfy a best fit line, then the data points do NOT satisfy polynomial growth of the form $y = At^m$

When to use semi-log paper:

Suppose you suspect your data points satisfy exponential growth of the form $y = Ac^t$ for some constants A and c.

$y = Ac^t$
$log(y) = log(Ac^t)$
$log(y) = log(A) + tlog(c)$. Let $z = log(y)$. Then
$z = log(A) + tlog(c)$.

$z = [log(c)]t + log(A)$. I.e. we have the equation of a line where slope $= log(c)$ and z-intercept $= log(A)$.

If $z = mt + b$, then (i) $log(A) = b$. Hence $A = 10^{log(A)} = 10^b$. (ii) $log(c) = m$. Hence $c = 10^m$.

Hence to determine the constants A and c in $y = Ac^t$, graph (t, y) on semi-log paper (note this is the same as taking $z = log(y)$), and determine equation of best fit line, $z = mx + b$. Then $y = 10^b(10^m)^t$. I.e., $y = 10^b(10^{mt})$

However if the data points do not satisfy a best fit line, then the data points do NOT satisfy polynomial growth of the form $y = Ac^t$

Semi-log and log-log plots problems (not HW, but highly recommended).

For each of the data sets below, graph these points on either semi-log or log-log paper and determine the function which best models these data points from the choices below.

1.) (1, 10), (8, 40), (32, 100), (8000, 4000)
2.) (1, 10000), (2, 3900), (6, 620), (7, 290)
3.) (1, 10000), (5, 400), (15, 50), (73, 2)
4.) (0, 1), (1.4, 3), (4.4, 30), (8, 480)
5.) (0, 100), (2, 45), (3.2, 7), (4, 2)
6.) (1, 1), (60, 4), (200, 6), (3200, 15), (8000, 20)
7.) (1, 1000), (5, 200), (20, 50), (515, 2)
8.) (0, 10), (0.6, 40), (1.8, 605), (2, 1000)
9.) (1, 100), (35, 600), (400, 2000), (8100, 9000)

A) $y = 0$ **B)** $y = t^{\frac{1}{3}}$ **C)** $y = t^{\frac{1}{2}}$ **D)** $y = t^{\frac{2}{3}}$ **E)** $y = 10^t$ **F)** $y = t^{\frac{3}{2}}$ **G)** $y = t^2$

H) $y = 1$ **I)** $y = t^{-\frac{1}{3}}$ **J)** $y = t^{-\frac{1}{2}}$ **K)** $y = t^{-\frac{2}{3}}$ **L)** $y = t^{-1}$ **M)** $y = t^{-\frac{3}{2}}$ **N)** $y = t^{-2}$

0) $y = 10$ **P)** $y = 10t^{\frac{1}{3}}$ **Q)** $y = 10t^{\frac{1}{2}}$ **R)** $y = 10t^{\frac{2}{3}}$ **S)** $y = 10t$ **T)** $y = 10t^{\frac{3}{2}}$ **U)** $y = 10t^2$

V) $y = 10t^{-\frac{1}{3}}$ **W)** $y = 10t^{-\frac{1}{2}}$ **X)** $y = 10t^{-\frac{2}{3}}$ **Y)** $y = 10t^{-1}$ **Z)** $y = 10t^{-\frac{3}{2}}$ **ZZ)** $y = 10t^{-2}$

a) $y = 100$ **b)** $y = 100t^{\frac{1}{3}}$ **c)** $y = 100t^{\frac{1}{2}}$ **d)** $y = 100t^{\frac{2}{3}}$ **e)** $y = 100t$ **f)** $y = 100t^{\frac{3}{2}}$ **g)** $y = 100t^2$

h) $y = 100t^{-\frac{1}{3}}$ **i)** $y = 100t^{-\frac{1}{2}}$ **j)** $y = 100t^{-\frac{2}{3}}$ **k)** $y = 100t^{-1}$ **l)** $y = 100t^{-\frac{3}{2}}$ **m)** $y = 100t^{-2}$

n) $y = 1000t^{\frac{1}{3}}$ **o)** $y = 1000t^{\frac{1}{2}}$ **p)** $y = 1000t^{\frac{2}{3}}$ **q)** $y = 1000t$ **r)** $y = 1000t^{\frac{3}{2}}$ **s)** $y = 1000t^2$

t) $y = 1000t^{-\frac{1}{3}}$ **u)** $y = 1000t^{-\frac{1}{2}}$ **v)** $y = 1000t^{-\frac{2}{3}}$ **x)** $y = 1000t^{-1}$ **y)** $y = 1000t^{-\frac{3}{2}}$ **z)** $y = 1000t^{-2}$

B) $y = 10^{\frac{t}{3}}$ **C)** $y = 10^{\frac{t}{2}}$ **D)** $y = 10^{\frac{2t}{3}}$ **E)** $y = 10(10^t$ **F)** $y = 10^{\frac{3t}{2}}$ **G)** $y = 10^{2t}$

I) $y = 10^{-\frac{t}{3}}$ **J)** $y = 10^{-\frac{t}{2}}$ **K)** $y = 10^{-\frac{2t}{3}}$ **L)** $y = 10^{-t}$ **M)** $y = 10^{-\frac{3t}{2}}$ **N)** $y = 10^{-2t}$

P) $y = 10(10^{\frac{t}{3}})$ **Q)** $y = 10(10^{\frac{t}{2}})$ **R)** $y = 10(10^{\frac{2t}{3}})$ **S)** $y = 10(10^t)$ **T)** $y = 10(10^{\frac{3t}{2}})$ **U)** $y = 10(10^{2t})$

W) $y = 10(10^{-\frac{t}{2}})$ **X)** $y = 10(10^{-\frac{2t}{3}})$ **Y)** $y = 10(10^{-t})$ **Z)** $y = 10(10^{-\frac{3t}{2}})$ **ZZ)** $y = 10(10^{-2t})$

c) $y = 100(10^{\frac{t}{2}})$ **d)** $y = 100(10^{\frac{2t}{3}})$ **e)** $y = 100(10^t)$ **f)** $y = 100(10^{\frac{3t}{2}})$ **g)** $y = 100(10^{2t})$

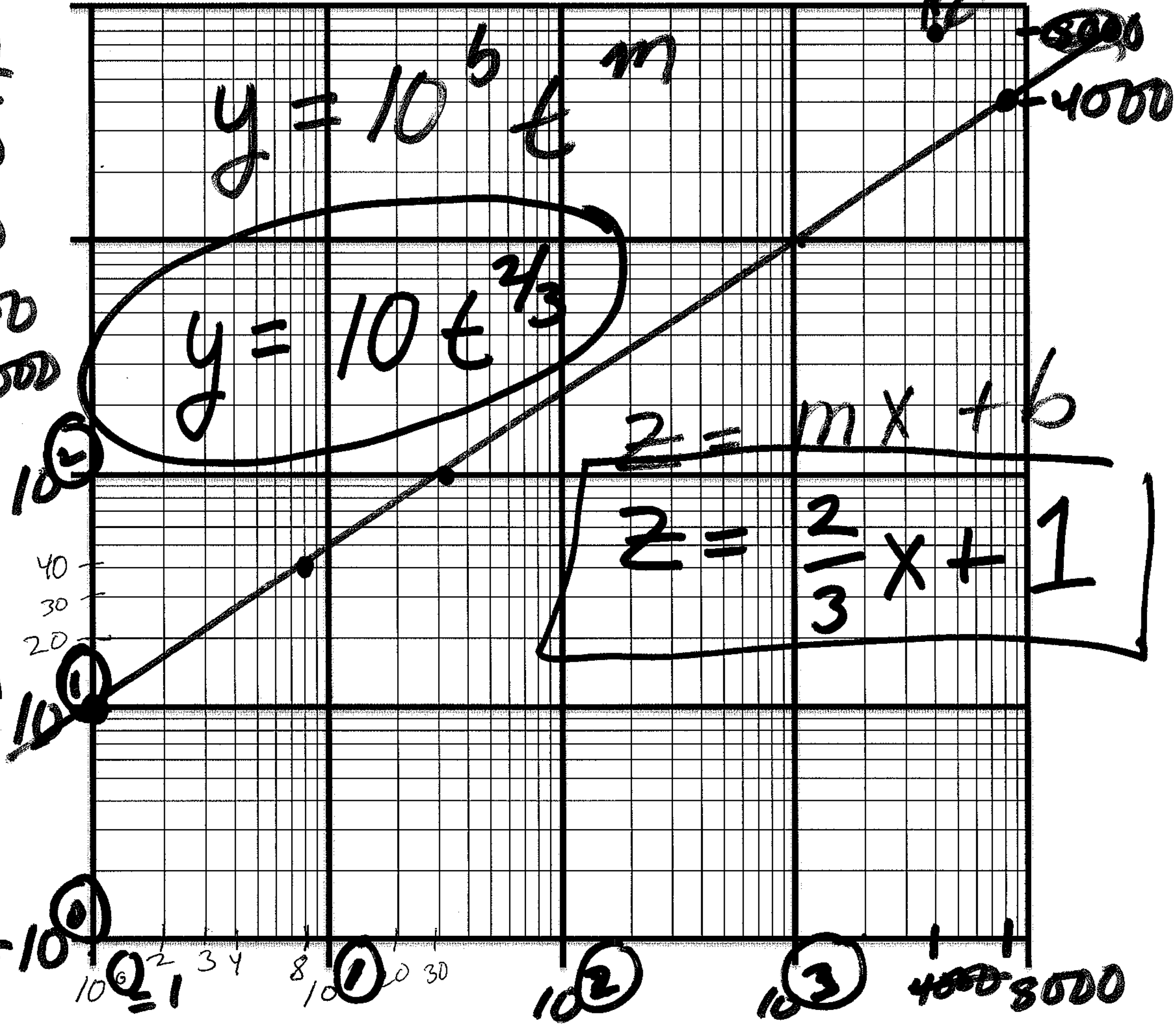
i) $y = 100(10^{-\frac{t}{2}})$ **j)** $y = 100(10^{-\frac{2t}{3}})$ **k)** $y = 100(10^{-t})$ **l)** $y = 100(10^{-\frac{3t}{2}})$ **m)** $y = 100(10^{-2t})$

o) $y = 1000(10^{\frac{t}{2}})$ **p)** $y = 1000(10^{\frac{2t}{3}})$ **q)** $y = 1000(10^t)$ **r)** $y = 1000(10^{\frac{3t}{2}})$ **s)** $y = 1000(10^{2t})$

u) $y = 1000(10^{-\frac{t}{2}})$ **v)** $y = 1000(10^{-\frac{2t}{3}})$ **x)** $y = 1000(10^{-t})$ **y)** $y = 1000(10^{-\frac{3t}{2}})$ **z)** $y = 1000(10^{-2t})$

t	y
1	10
8	40
32	100
8000	4000

OOB
Darcy



$y = 10^{b} t^{m}$

$y = 10 t^{2/3}$

$z = mx + b$

$\boxed{z = \frac{2}{3} x + 1}$