5.9) post ex: definite integral

$$\int_0^b e^{-x}\,dx = -e^{-x}\Big|_0^b$$

$$= -e^{-b} - (-e^{-0})$$

$$= 1 - e^{-b}$$

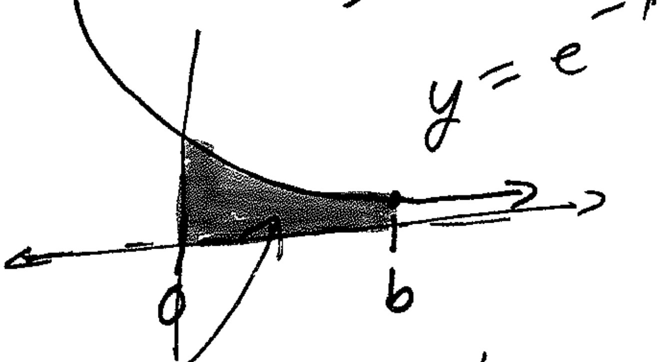
= area btwn

$y = 0$ (x-axis)

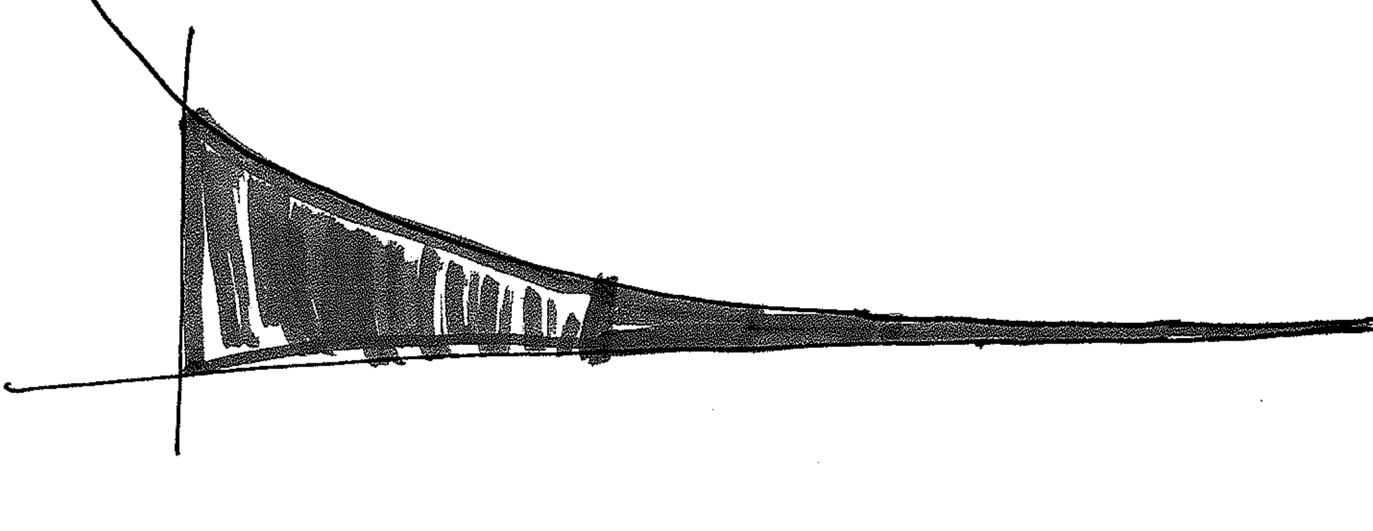
$y = e^{-x}$

$x = 0$

$x = b$



Note this area $= 1 - e^{-b} < 1$

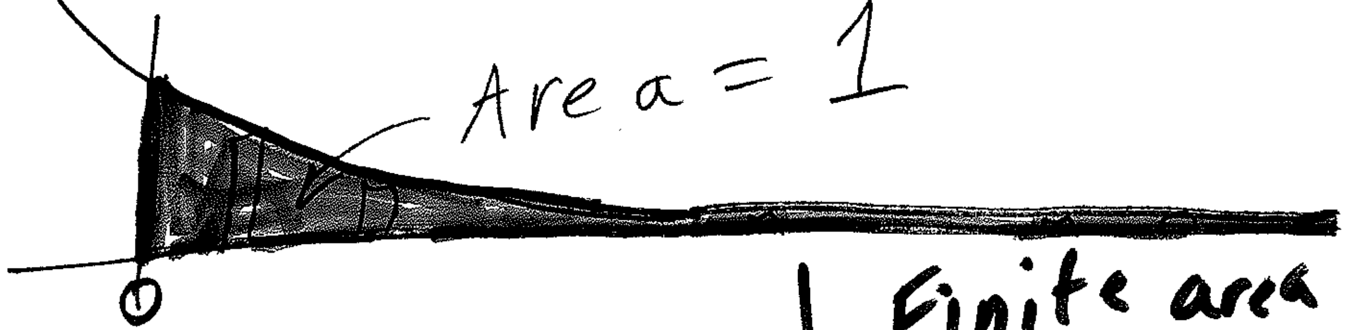


Find area betwee

$y = 0$ $\qquad$ $y = e^{-x}$

(x-axis

and $\quad x \geq 0$



$$= \int_0^\infty e^{-x}\,dx$$

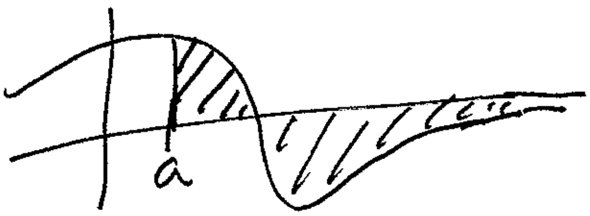
Finite area makes sense since $\int_0^b e^{-x} < 1$

$$= \lim_{b \to \infty} \int_0^b e^{-x}\,dx$$

$$= \lim_{b \to \infty} \left[1 - \underbrace{e^{-b}}_{\to\, 0} \right] = 1$$

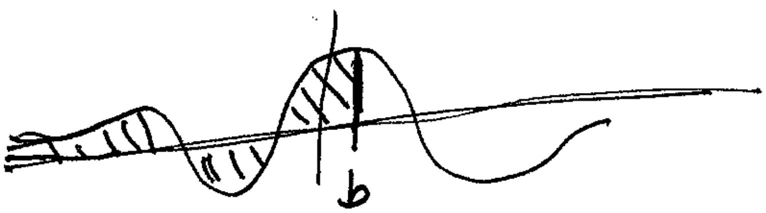
$$\int_a^\infty f(x)\,dx = \lim_{b\to\infty} \int_a^b f(x)\,dx$$

= net area btwn $y = f(x)$
$y = 0$ (x-axis)
where $x \geq a$



$$\int_{-\infty}^b f(x)\,dx = \lim_{a\to-\infty} \int_a^b f(x)\,dx$$

= net area btwn $y = f(x)$
$y = 0$ (x-axis)
where $x \leq b$



$$\int_{-\infty}^\infty f(x)\,dx = \int_{-\infty}^a f(x)\,dx + \int_a^\infty f(x)\,dx$$

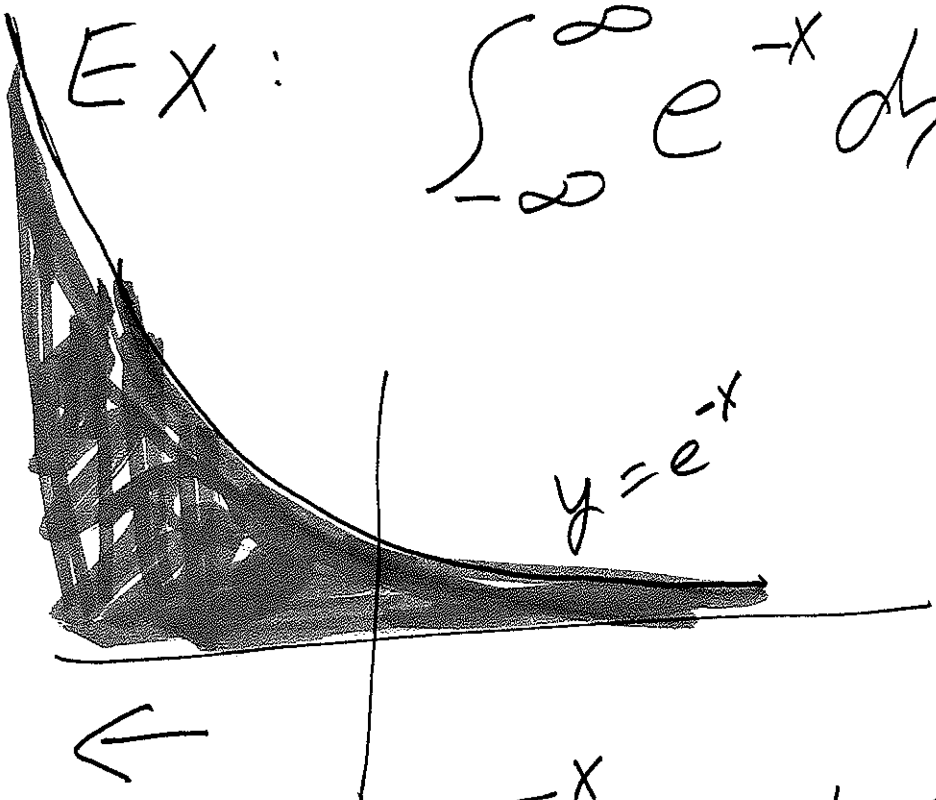
~~net area~~

Definitions of Improper Integral

Ex: $\int_{-\infty}^{\infty} e^{-x}\,dx = +\infty$

DNE

DIVERGENT



height $= e^{-x} \to +\infty \Rightarrow$ area $\to +\infty$

A.) DNE (Divergent)

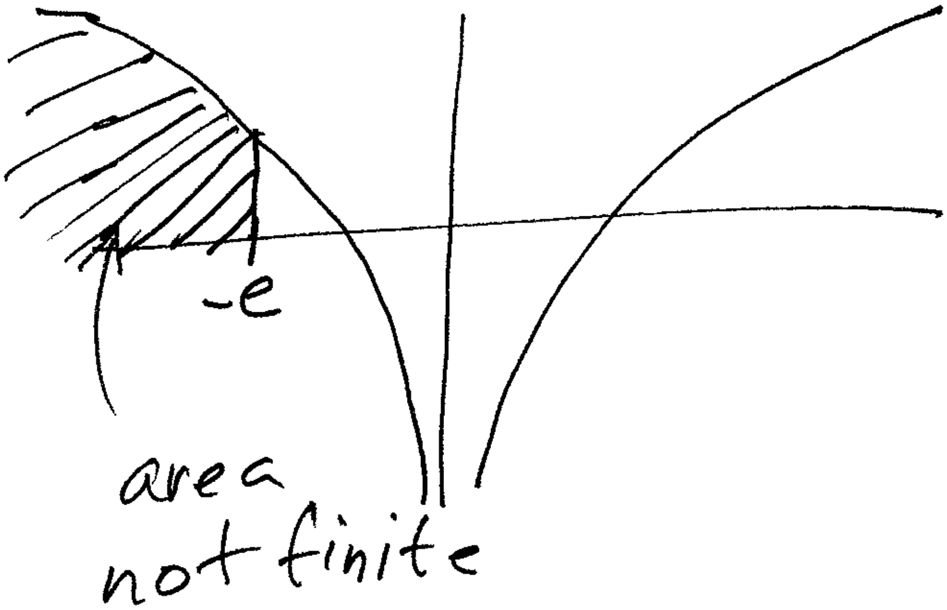
$$\int_{-\infty}^{\infty} e^{-x}\,dx = \int_{-\infty}^{0} e^{-x}\,dx + \int_{0}^{\infty} e^{-x}\,dx$$

$= \_\_\_\_\_$

can do via algebra but why waste time when answer is obvious

DNE

Ex: $\int_{-\infty}^{-e} \ln|x|\,dx = \text{DIV}$
(DNE)



$$\int_{-100}^{\infty} \underbrace{x^3 e^{\sqrt{x}}}_{\text{height}}\,dx = \text{DIV}$$

height of rectangle $= x^3 e^{\sqrt{x}} \longrightarrow +\infty$

as $x \to +\infty$

so area not finite

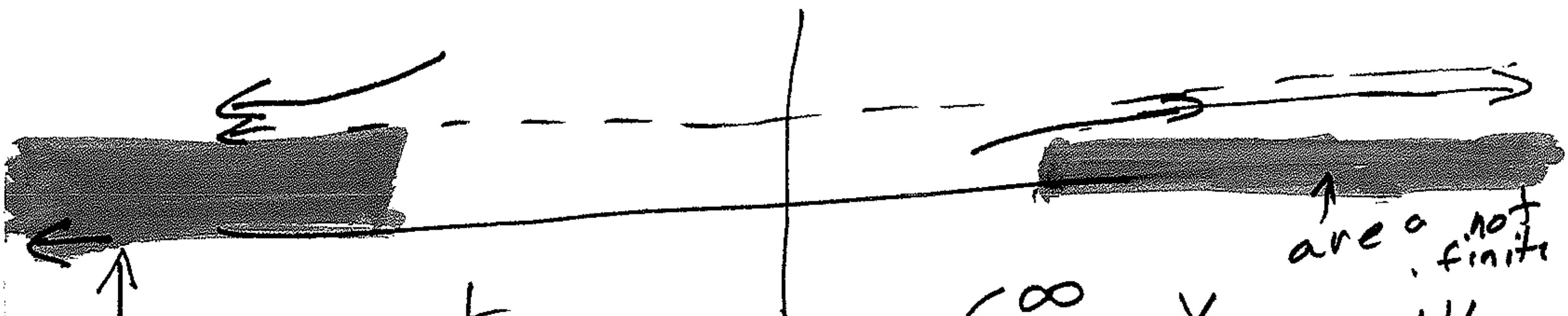
area $\to +\infty$

$$\int_{-\infty}^{10000} \frac{x}{1+x}\, dx = \text{DIV (DONE)}$$

$$\text{height} = \frac{x}{1+x} \longrightarrow 1$$

as $x \to -\infty$

(eg: $\frac{-100}{-99}$, $\frac{x}{1+x} \sim \frac{x}{x} = 1$ for large x

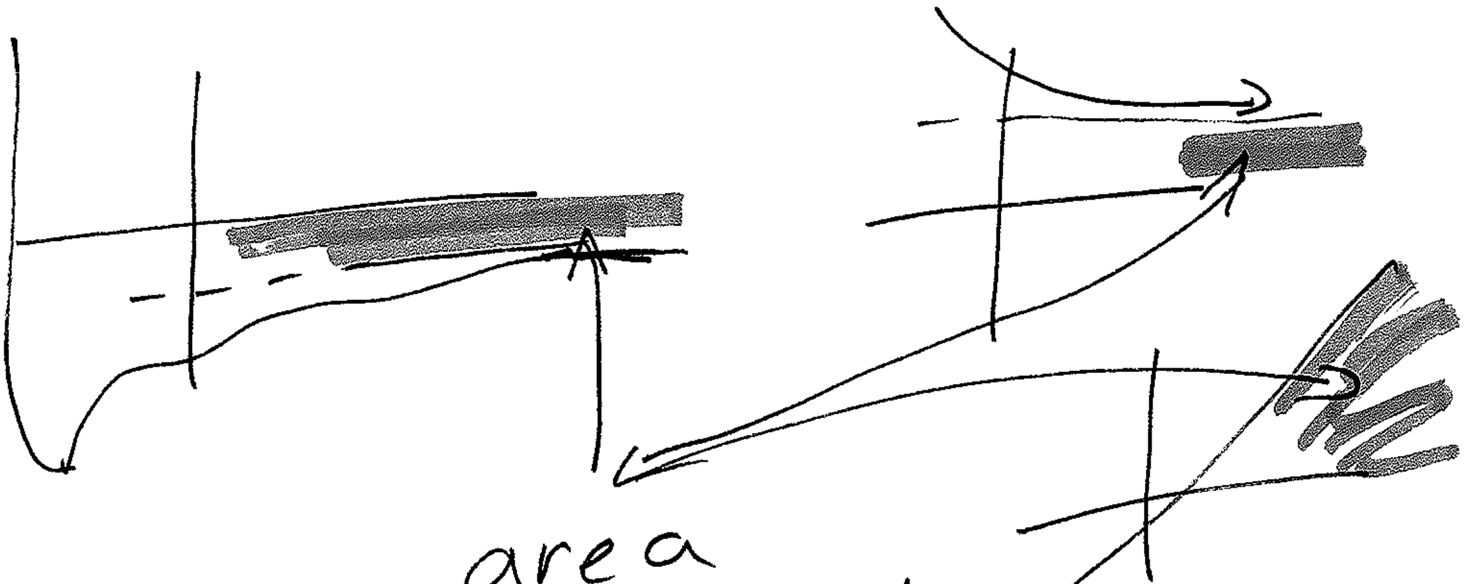


area not finite

$$\int_{-1000}^{\infty} \frac{x}{1+x} \quad \text{DIV}$$

$$\lim_{x \to +\infty} f(x) \neq 0 \Rightarrow \int_a^{\infty} f(x)\,dx \text{ DIV}$$

DNE



area
NOT finite

$$\lim_{x \to -\infty} f(x) \neq 0 \Rightarrow \int_{-\infty}^{b} f(x)\,dx$$

DIV
(DNE)

$$\lim_{x \to +\infty} f(x) = 0 \Rightarrow \int_a^{+\infty} f(x)\,dx = ?$$

$-\infty$ ← similar

$$\int_{-\infty}^{100} \frac{x\,dx}{1+x^2}$$

so need to calc.

$$\text{height} = \frac{x}{1+x^2} \approx \frac{x}{x^2} \approx \frac{1}{x} \to 0$$

as $x \to -\infty$

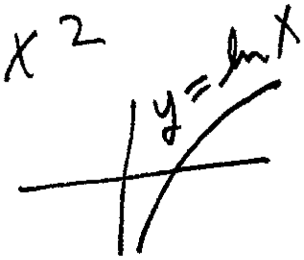
for large ~~#~~ negative #'s

$$\frac{1}{2}\int \frac{2x\,dx}{1+x^2} = \frac{1}{2}\int \frac{du}{u} = \frac{1}{2}\ln|u|$$

$$= \frac{1}{2}\ln(1+x^2)$$

$$\boxed{\begin{aligned} \text{Let } u &= 1+x^2 \\ du &= 2x\,dx \end{aligned}}$$

$$\int_{-\infty}^{100} \frac{x\,dx}{1+x^2} = \lim_{a\to-\infty} \int_a^{100} \frac{x\,dx}{1+x^2}$$



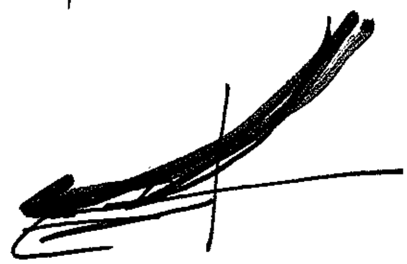
$$\lim_{a\to-\infty} \frac{1}{2}\ln(1+x^2)\Big|_a^{100}$$

$$= \lim_{a\to-\infty}\left[\frac{1}{2}\ln 10{,}001 - \frac{1}{2}\overbrace{\ln(1+a^2)}^{+\infty}\right] = \text{DIV}$$

(DONE)

Ex $\int_{-\infty}^{0} e^{3x}\sqrt{2-e^{3x}}\,dx$

$x \to -\infty \Rightarrow e^{x} \to 0$



height $= e^{3x}\sqrt{2-e^{3x}} \to 0$

Let $u = 2 - e^{3x}$

$$\frac{du}{-3} = \frac{-\cancel{3}e^{3x}\,dx}{-\cancel{3}}$$

$$\int \sqrt{2-e^{3x}}\left(e^{3x}dx\right) = \int u^{1/2}\left(\frac{du}{-3}\right)$$

$$= \frac{-2}{9}u^{3/2} = -\frac{2}{9}\left(2-e^{3x}\right)^{3/2}$$

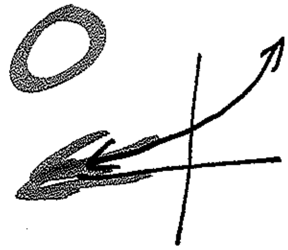
$$\int_{-\infty}^{0} e^{3x}\sqrt{2-e^{3x}}\,dx$$

$$= \lim_{a\to -\infty} \int_{a}^{0} \left[e^{3x}\sqrt{2-e^{3x}}\right] dx$$

$$= \lim_{a\to -\infty} -\frac{2}{9}\left(2-e^{3x}\right)^{3/2}\Big|_{a}^{0}$$

$$= \lim_{a\to -\infty} \left[-\frac{2}{9}\left(2-e^{0}\right)^{3/2} - \left[-\frac{2}{9}\left(2-e^{3a}\right)^{3/2}\right]\right]$$

$$= \lim_{a\to -\infty} \left[-\frac{2}{9} + \frac{+2}{9}\left(2-\underbrace{e^{3a}}_{0}\right)^{3/2}\right]$$



$$= -\frac{2}{9} + \frac{+2}{9}(2)^{3/2}$$

$$= \frac{-2+4\sqrt{2}}{9} = \frac{-2+4\sqrt{2}}{9}$$