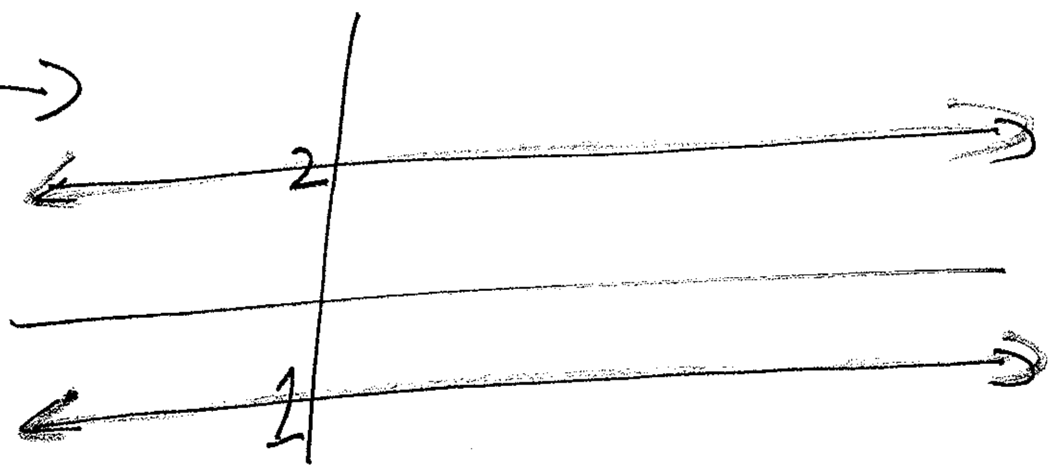
8.3: $y' = f(y)$

autonomous diff eq'n

Ex: $y' = ky$

$y' = (2-y)(y+1)$

$y' = \sin y$



$y' = 0 \Rightarrow y = 2, -1$

Defn: An equilibrium sol'n is a constant sol'n to the diff equation
(value)

$$y = C \implies y' = 0$$

C ↑ constant

$\longleftrightarrow y = c$

Ex: $y' = \sin y$

$y' = 0 = \sin y$

$y = 0,\ y = \pi,\ y = -\pi, \ldots$

$y = k\pi$ for any integer K

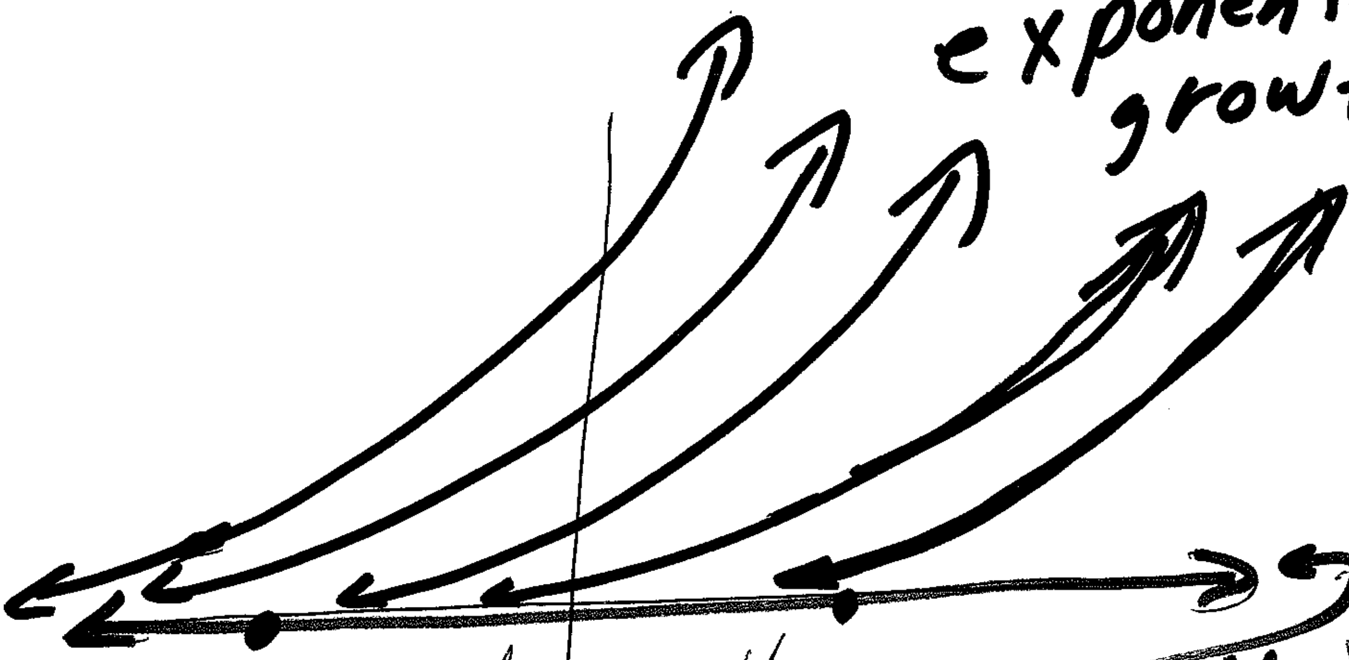
EX: $y' = 2y$

Equil: $y' = 0 = 2y$

$\Rightarrow \boxed{y = 0}$ ← equil sol'n

Recall $y' = 2y \Rightarrow y = Ce^{2t}$

exponential growth



exponential growth (except equil soln)

equil $y = 0$ $(c = 0)$

initial value $y(t_0) = 0$

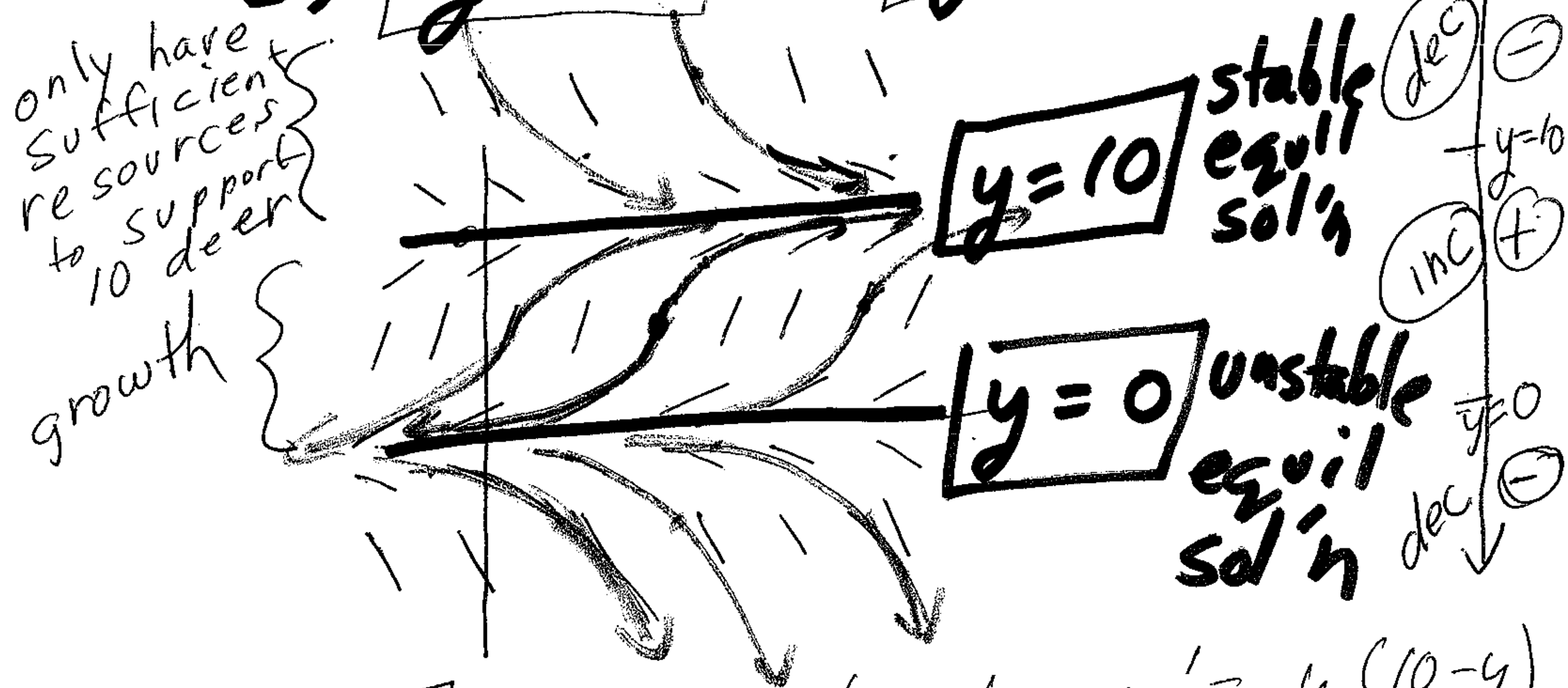
models population growth

$$y' = y(10-y) \Leftarrow \text{logistic growth}$$

Equil sol'n:

$$y' = 0 = y(10-y)$$

$$\Rightarrow \boxed{y=0}, \boxed{y=10}$$



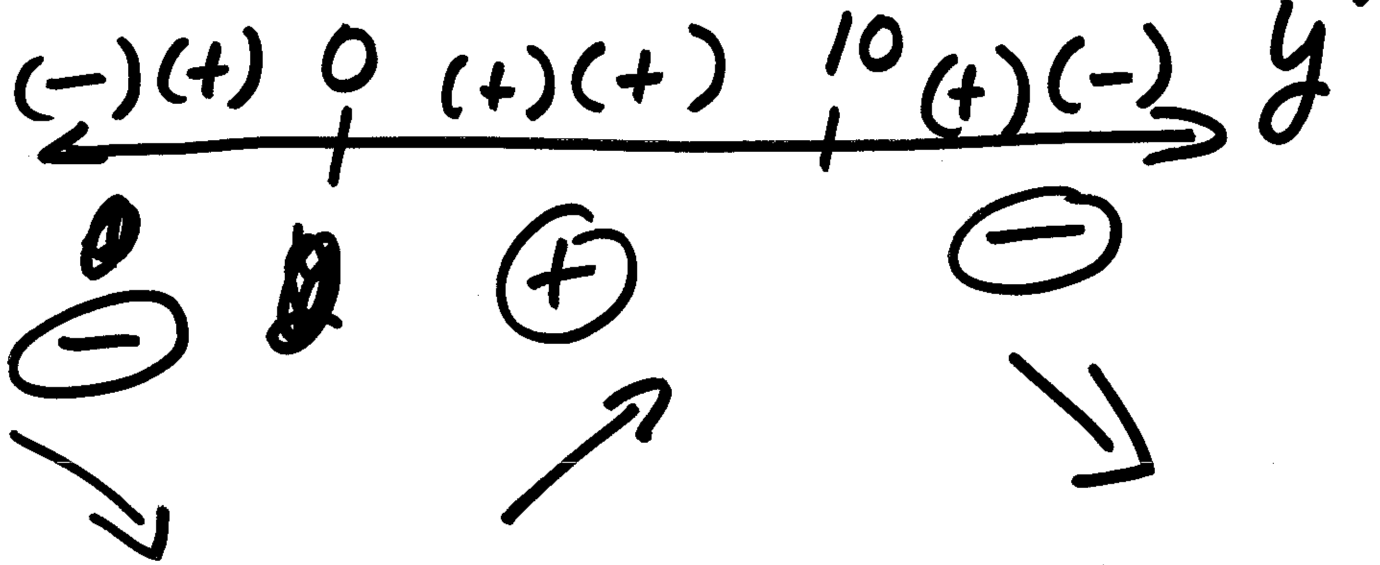
Note: $\boxed{y=10}$ is a soln to $y' = y(10-y)$

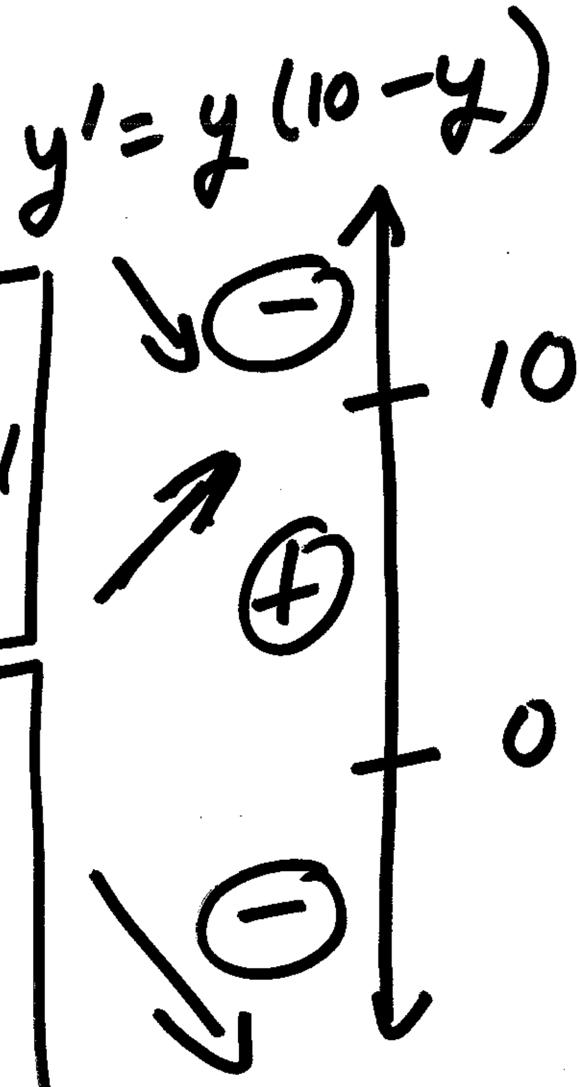
$$\left.\begin{array}{l} y' = 0 \\ 10(10-10) = 0 \end{array}\right\} = \Rightarrow y=10 \text{ is a sol'n}$$

Logistic growth model:

$$y' = ky\left(1 - \frac{y}{L}\right), \quad k>0, \ L>0$$

$y' = y(10-y)$





$\boxed{y=10 \text{ is a stable equil sol'n}}$

$\boxed{y=0 \text{ is an unstable equil sol'n}}$

Defn: An equilibrium sol'n $y=c$ is stable if

if initial value starts close to $y=c$

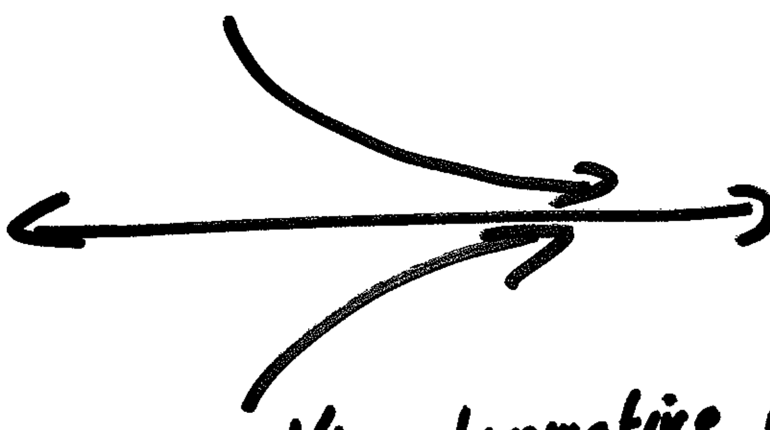
($y(t_0) = L$ where L is close to c

then sol'n approaches $y=c$

---

or equiv $y=c$ is a stable equil sol'n

if



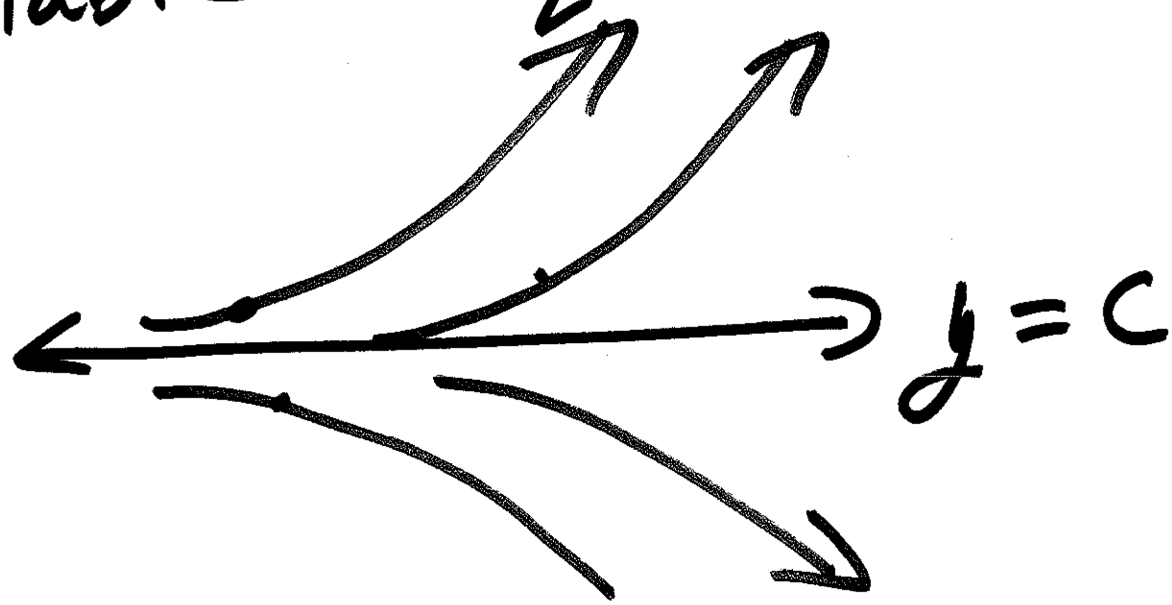
$y=c$

---

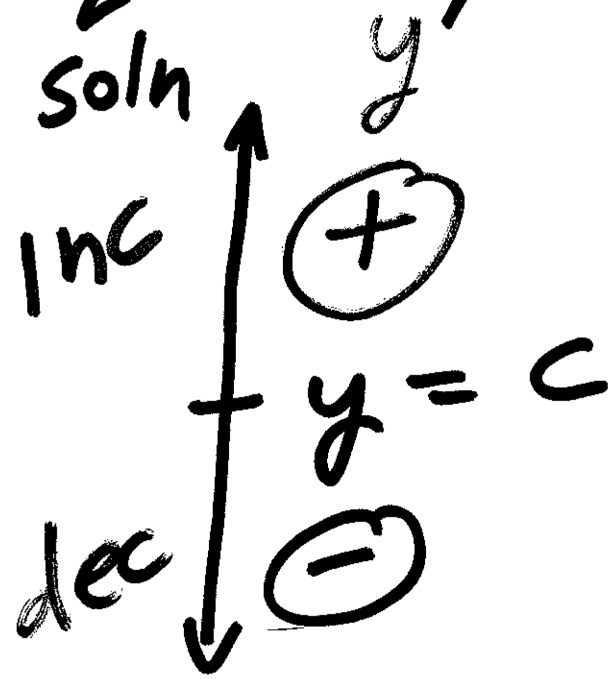
or equiv

sol'n	derivative y'
dec	$-$
$y = c$	
inc	$+$

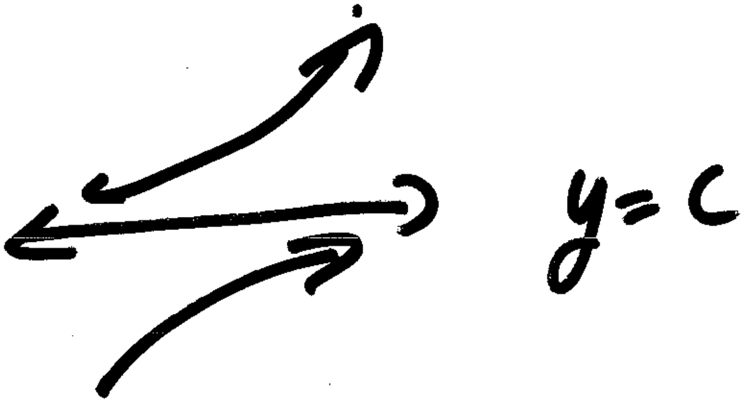
Defn: $y = c$ is an unstable equil soln if

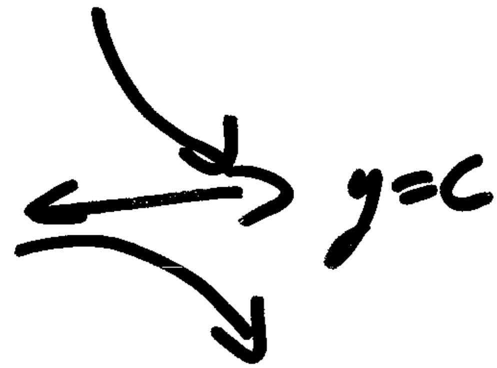


or equivalently

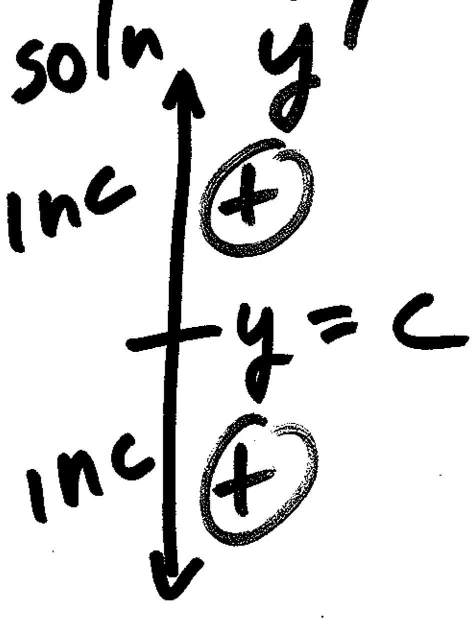


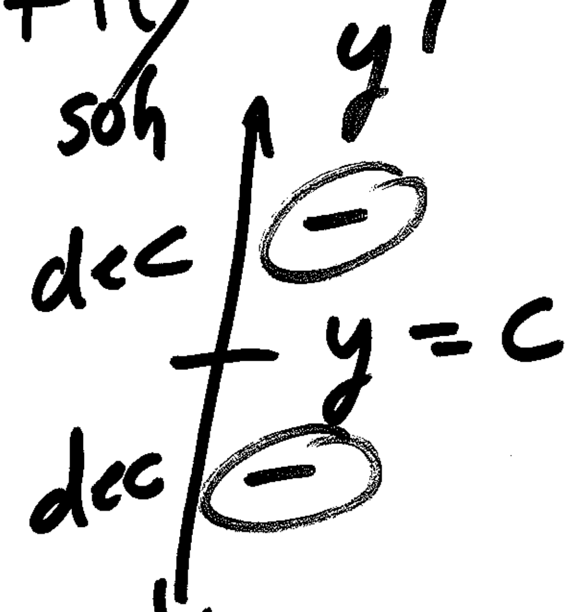
Defn: $y = c$ is a semi-stable equil soln if





or equivalently





Logistic Growth w/ a Threshold

$$y' = ky\left(1-\frac{y}{L}\right)\left(\frac{y}{T}-1\right)$$

$$k>0\ , \quad L>T>0$$

Equilibrium sol'n

$$y' = 0 = ky\left(1-\frac{y}{L}\right)\left(\frac{y}{T}-1\right)$$

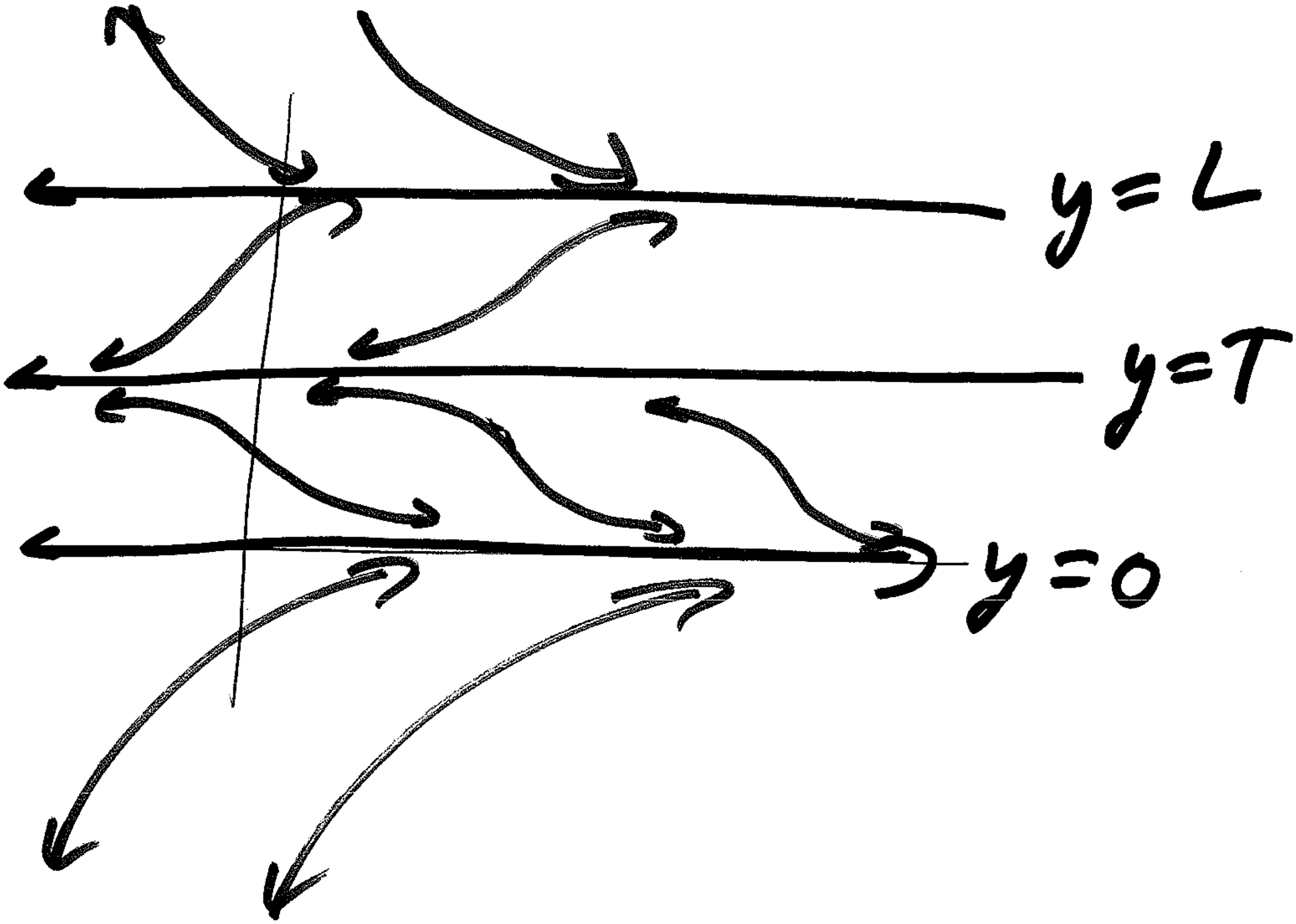
$$\Rightarrow \boxed{y=0\ ,\ y=L\ ,\ y=T}$$

soln		$y' = ky\left(1-\frac{y}{L}\right)\left(\frac{y}{T}-1\right)$
$-$		$(+)(-)(+)$
	L	
$+$		$(+)(+)(+)$
	T	
$-$		$(+)(+)(-)$
	0	
$+$		$(-)(+)(-)$

y = L is a stable eq soln

y = T is a unstable eq soln

y = 0 is a stable eq. soln



y=L
y=T
y=0