5.7 $$\int \frac{1}{\sqrt{x^2 \pm a^2}}\,dx = \ln\left|x + \sqrt{x^2 \pm a^2}\right|$$

$$\int \frac{1}{\sqrt{2x^2+3}}\,dx$$

$$= \int \frac{1}{\sqrt{2\left(x^2 + 3/2\right)}}\,dx = \int \frac{dx}{\sqrt{2}\,\sqrt{x^2+\frac{3}{2}}}$$

$$= \frac{1}{\sqrt{2}} \int \frac{dx}{\sqrt{x^2 + \left(\sqrt{\frac{3}{2}}\right)^2}}$$

$$a = \sqrt{\frac{3}{2}} \Rightarrow \boxed{a^2 = \frac{3}{2}}$$

$$= \frac{1}{\sqrt{2}} \ln\left|x + \sqrt{x^2 + \frac{3}{2}}\right| + C$$

$$\int \frac{1}{\sqrt{2x^2-3}}\,dx$$

$$= \int \frac{1}{\sqrt{2\left(x^2-\frac{3}{2}\right)}}\,dx$$

$$= \frac{1}{\sqrt{2}} \int \frac{1}{\sqrt{x^2-\frac{3}{2}}}\,dx \qquad a^2 = \frac{3}{2}$$

$$= \boxed{\frac{1}{\sqrt{2}} \ln\left| x + \sqrt{x^2 - \frac{3}{2}} \right| + C}$$

/2

$$\int \frac{1}{\sqrt{3-2x^2}}\,dx$$

$$\boxed{\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right)}$$

$$\int \frac{1}{\sqrt{2\left(\frac{3}{2}-x^2\right)}}\,dx$$

$$a^2 = \frac{3}{2}$$

$$= \frac{1}{\sqrt{2}}\int \frac{1}{\sqrt{\frac{3}{2}-x^2}}\,dx$$

$$\Rightarrow a = \sqrt{\frac{3}{2}}$$

$$= \boxed{\frac{1}{\sqrt{2}}\left(\sin^{-1}\left(\sqrt{\frac{2}{3}}\,x\right)\right) + C}$$

8.1 : Differential eqns

$$f'(x) = \cos x$$

$$\frac{dy}{dx} = \cos x$$

$$y' = \cos x$$

$$\boxed{y = \sin x + C}$$

general sol'n

---

longer: $\frac{dy}{dx} = \cos x \Rightarrow dy = \cos x \, dx$

$$\int dy = \int \cos x \, dx \Rightarrow y = \sin(x) + C$$

Initial value problems

$$f'(x) = \cos x, \quad f\left(\frac{\pi}{2}\right) = 3$$

$$f(x) = \sin x + C$$

$$y' = \cos x, \quad y\left(\frac{\pi}{2}\right) = 3$$

$$y = \sin x + C$$

$y\left(\frac{\pi}{2}\right) = 3$: $\quad 3 = \sin\left(\frac{\pi}{2}\right) + C$

$$3 = 1 + C \Rightarrow C = 2$$

$$\boxed{y = \sin x + 2}$$

soln to IVP

$\boxed{\text{IVP}: y'' = 2, \quad \underline{y'(0) = 1}, \quad \underline{y(2) = 9}}$

$$y' = 2x + C$$

$y'(0) = 1$: $\quad 1 = 2(0) + C \Rightarrow C = 1$

$$y' = 2x + 1$$

$$y = x^2 + x + C$$

← different C
Bad abuse of notation

$y(2) = 9$: $\quad 9 = 2^2 + 2 + C$

$$9 = 6 + C \Rightarrow C = 3$$

$$\boxed{y = x^2 + x + 3}$$ ← IVP sol'n

better method

$y'' = 2$

$y' = 2x + C$ ← $y'(0) = 1$

$y(2) = 9$

$\boxed{y = x^2 + Cx + k}$ ← $y(2) = 9$

← general sol'n

Weird example

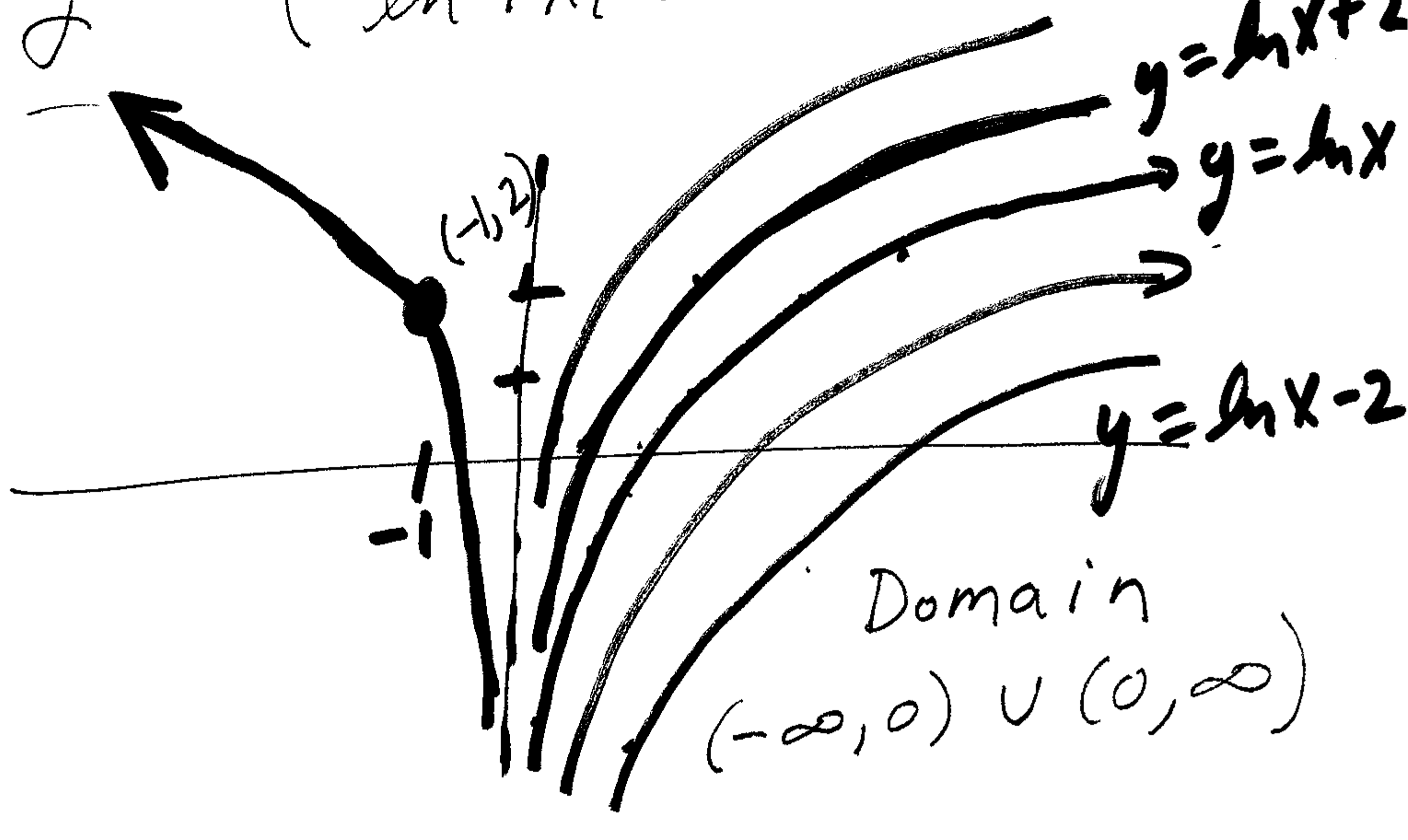
IVP: $y' = \frac{1}{x} \quad y(-1) = 2$

$$y = \ln|x| + C$$

$y(-1)=2$: $\quad 2 = \cancelto{0}{\ln|-1|} + C$

$$\Rightarrow C = 2$$

$$y = \begin{cases} \ln|x| + 2 & \text{if } x<0 \\ \ln|x| + C & \text{if } x>0 \end{cases}$$



Domain

$(-\infty, 0) \cup (0, \infty)$

Normally

$$y' = f'(x) \quad , \quad y(x_0) = y_0$$

$\Rightarrow$ unique sol'n

$$y = f(x) + C$$

$$y_0 = f(x_0) + C$$

$$C = y_0 - f(x_0)$$

$$\boxed{y = f(x) + (y_0 - f(x_0))}$$

But this assume
domain connected

EX: $\sqrt{3+(y'')^2-(\sin x)y'}=2$

eqn(*)

Show $y=\cos x$ is a sol'n to (*)

$$y' = -\sin x$$
$$y'' = -\cos x$$

$\sqrt{3+(y'')^2-(\sin x)y'}$

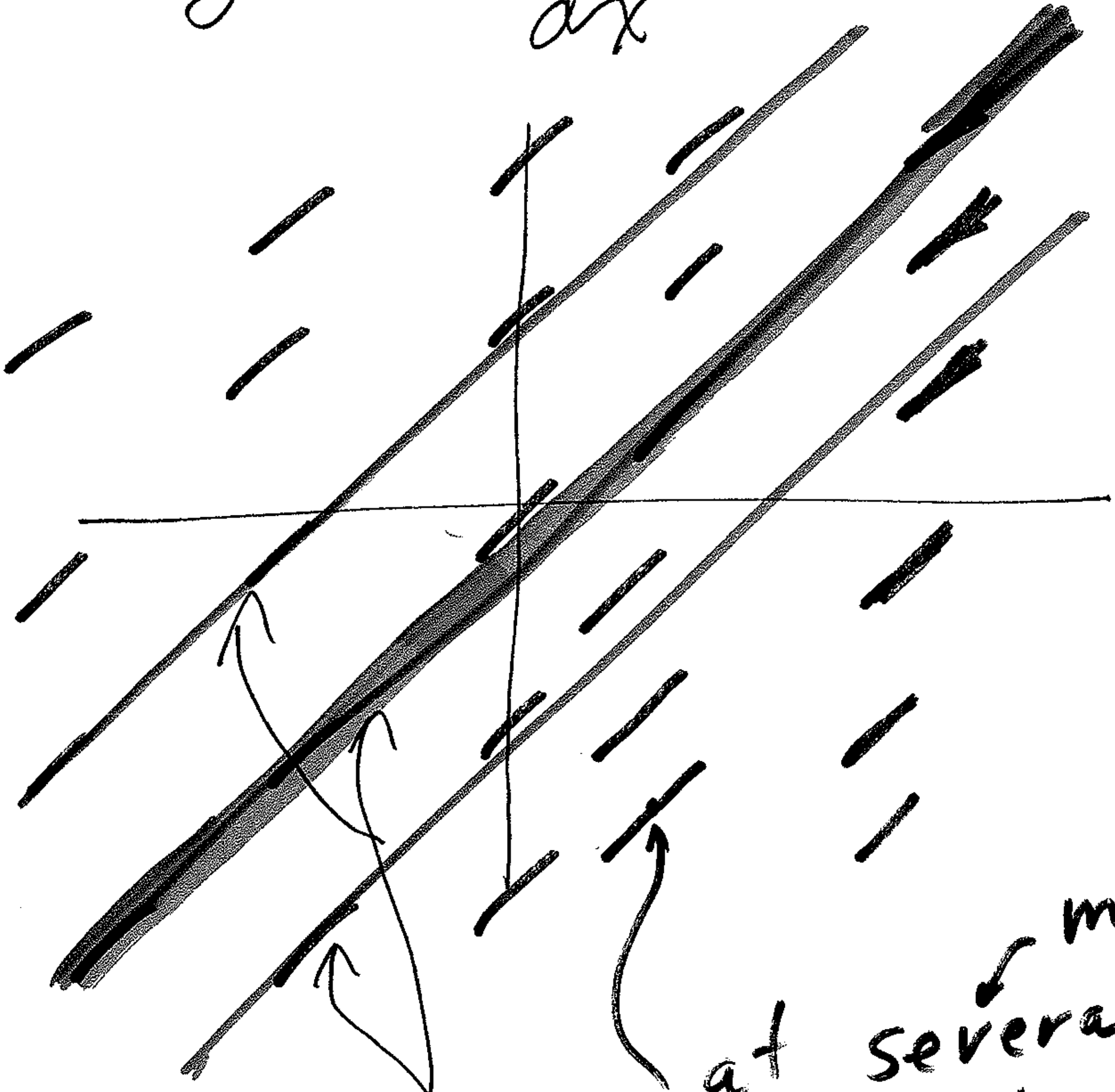
$=\sqrt{3+(-\cos x)^2-(\sin x)(-\sin x)}$

$=\sqrt{3+(\cos^2 x+\sin^2 x)}$

$=\sqrt{3+1}=\sqrt{4}=2$ ✓

Slope fields

$$y' = \frac{dy}{dx} = 1$$



$y' = 1$

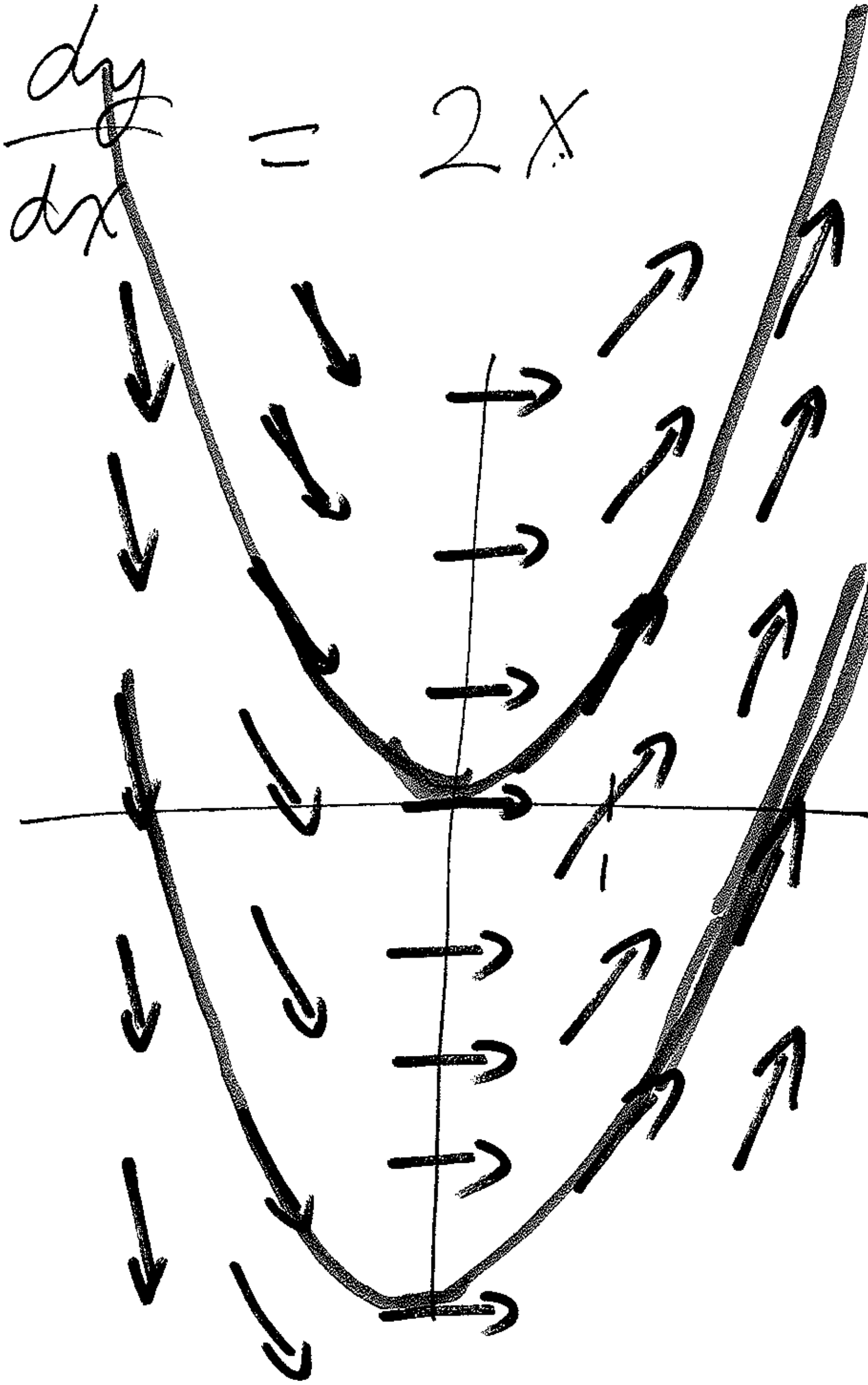
$y = x + C$

many

at several points
draw slope of
tangent line
(in reality, draw
small part of tangent
line

$$\frac{dy}{dx} = 2x$$

x	slope
0	0
1	$2(1) = 2$
-1	-2
2	4
-2	-4



$$y' = 2x \Rightarrow y = x^2 + C$$

$$\frac{dy}{dx} = \frac{x}{y}$$

$x=0 \Rightarrow$

slope $= \frac{0}{y} = 0$

$x=1 \Rightarrow$

slope $= \frac{1}{y}$

$x=y \Rightarrow$

slope

$= \frac{x}{x} = 1$