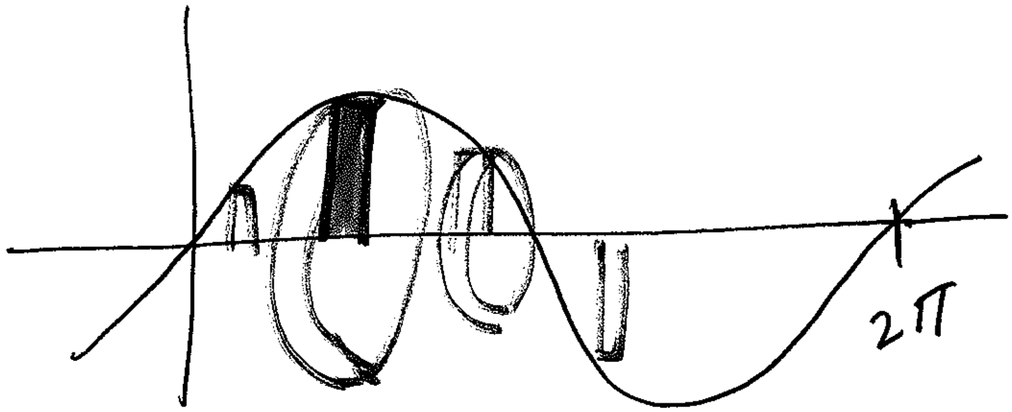
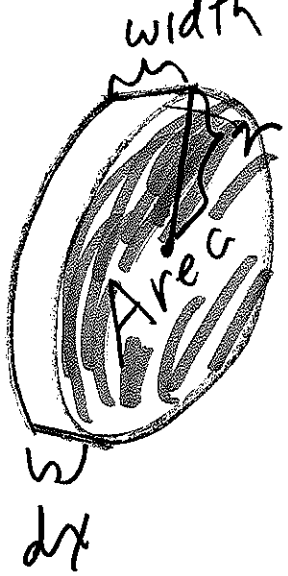
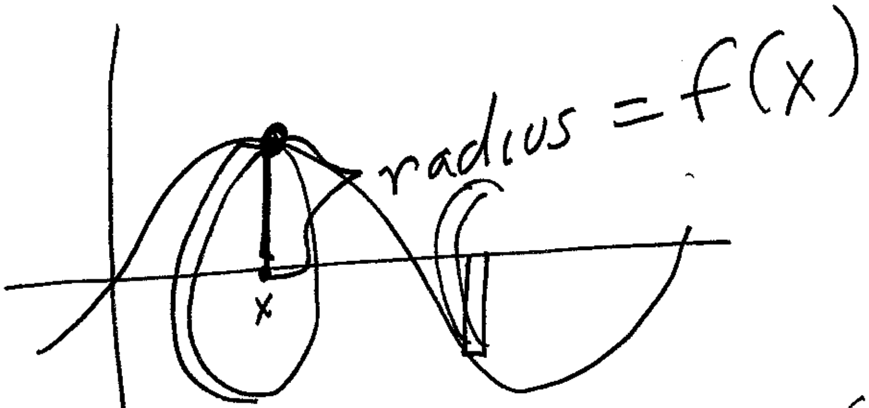




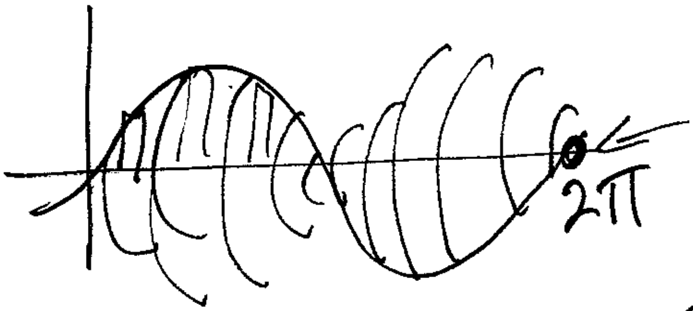
Volume of Disk $=$ Area $\cdot$ width

$\pi r^2 \cdot dx$



radius squared so don't care if f(x) is neg or pos

Volume of disk $= \pi \left(f(x)\right)^2 dx$



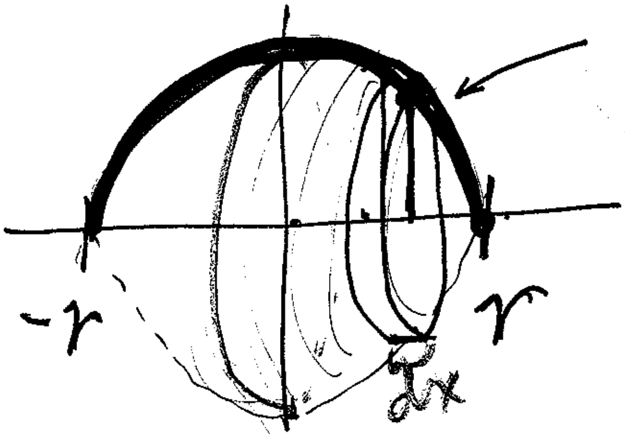
$y = \sin x$

since cross-section = circle

Volume of object obtained by rotating above $= \displaystyle\int_0^{2\pi} \pi \left(f(x)\right)^2 \, dx$

$$= \boxed{\int_0^{2\pi} \pi \sin^2 x \, dx}$$

Find the volume of a sphere of radius r



$x^2 + y^2 = r^2,\ y > 0$

$y = \sqrt{r^2 - x^2}$

$$\int_{-r}^{r} \overbrace{\pi \left(\sqrt{r^2 - x^2}\right)^2}^{\text{Area}} \overbrace{dx}^{\text{width}}$$

$$= 2 \int_0^r \pi \left(r^2 - x^2\right) dx$$

$$= 2\pi \left(r^2 x - \frac{x^3}{3}\right) \Big|_0^r$$

$$= 2\pi \left(r^3 - r^3/3\right) - 0$$

$$= \frac{4\pi r^3}{3}$$

/3

5.6 Integration by Parts

$$\int \underbrace{u}\,\underbrace{dv} = uv - \int v\,du.$$

$$\int \underbrace{x}_{u}\,\underbrace{e^x\,dx}_{dv}$$

$$= xe^x - \int e^x\,dx$$

$$= \boxed{xe^x - e^x + C}$$

Let $u = x$

$du = dx$

Let $dv = e^x\,dx$

$\int dv = \int e^x\,dx$

$v = e^x$

Product rule

$$\frac{d(u \cdot v)}{dx} = u \cdot \frac{dv}{dx} + v \frac{du}{dx}$$

$$\int \left[\frac{d(uv)}{dx}\right] dx = \int u \cdot \frac{dv}{\cancel{dx}} \cancel{dx} + \int v \frac{du}{\cancel{dx}} \cancel{dx}$$

$$uv = \int u\,dv + \int v\,du$$

$$\boxed{\int u \cdot dv = uv - \int v\,du}$$

Integration by parts
product rule for anti-derivative

$$uv \xrightarrow[\text{derivative}]{\text{take}} \frac{d(uv)}{dx} \xrightarrow[\text{deriv}]{\text{anti}} \int \frac{d(uv)}{dx}\,dx = uv$$

5.7

$$\int \frac{du}{a^2-u^2} = \frac{1}{2a} \ln\left|\frac{u+a}{u-a}\right| + C$$

$\boxed{a=\sqrt{3}}$

$$\boxed{\int \frac{dx}{3-x^2} = \frac{1}{2\sqrt{3}} \ln\left|\frac{x+\sqrt{3}}{x-\sqrt{3}}\right| + C}$$

$a^2 = 3 \Rightarrow a = \sqrt{3}$ ($a=-\sqrt{3}$ gives same answer) so redundant

$u = x$

$$\int \sec^2(4x)\,dx$$

Let $u = 4x$

$$\frac{du}{4} = \frac{\cancel{4}\,dx}{\cancel{4}}$$

$$= \int \frac{\sec^2(u)\,du}{4}$$

$$= \frac{\tan u}{4} + C$$

$$= \frac{\tan(4x)}{4} + C$$

$\int \sec^2 u\,du = \tan u + C$

$$\int e^{2u}\cos 3u\,du =$$

$$= \boxed{\frac{e^{2u}}{4+9}\left(2\cos(3u)+3\sin(3u)\right)+C}$$

$$\int e^{au}\cos(bu)\,du = \frac{e^{au}}{a^2+b^2}\left(a\cos(bu)+b\sin(bu)\right)+C$$

$$a=2\ ,\ b=3$$

$$\int e^{+ax^2}\,dx = \frac{-i\sqrt{\pi}}{2\sqrt{a}}\,\mathrm{erf}\left(i x\sqrt{a}\right)$$

$$\int e^{+3x^2}\,dx = \frac{-i\sqrt{\pi}}{2\sqrt{3}}\,\mathrm{erf}\left(i x\sqrt{3}\right)$$