To specify which \( y = ce^{kx} \)
Need to choose a point \((x_0, y_0)\)

Initial value problem
Solve \( \frac{dy}{dx} = ky \) where \( y(x_0) = y_0 \) (ie \((x_0, y_0)\) lies on graph of the sol'n)

There exists a unique sol'n to this I.V.P.
\[ y' = ky \]

**Pre-calculus**

**IVP**

\[
(y_0, y_0) \text{ lies on } y = Ce^{kx}
\]

\[ y_0 = Ce^{kx_0} \]

\[
\Rightarrow C = \frac{y_0}{e^{kx_0}}
\]
Solve Initial Value Problem

\[ \frac{dy}{dt} = 5y \quad \text{where} \quad y(0) = 3 \]

\[ y = Ce^{5t} \]

\[ 3 = Ce^0 \quad \Rightarrow \quad C = 3 \]

\[ y = 3e^{5t} \]

Check \[ y' = 15e^{5t} \]

\[ 5y = 15e^{5t} \quad \checkmark \]
Suppose invest $1000 at a rate of 3% per year compounded continuously.

a) How much after 1 year?

b) When will you double your money?

\[
\frac{dP}{dt} = 0.03P
\]

\[
\Rightarrow P = Ce^{0.03t}
\]

\[t=0, \quad 1000 = Ce^0 \Rightarrow c=1000\]

\[P = 1000e^{0.03t}\]

a) \[P(1) = 1000e^{0.03(1)} = 1030.45\]
b) Find \( t \) when \( P = 2000 \)

\[
2000 = 1000 \cdot e^{0.03t}
\]

\[
\ln 2 = \ln e^{0.03t}
\]

\[
\ln 2 = 0.03t
\]

\[
t = \frac{\ln 2}{0.03}
\]

\[
= \frac{0.69...}{0.03}
\]

\[
= 23.1...
\]

\[
\ln 2 = 0.69...
\]
Doubling time = generation time

= the time it takes to double initial amount

\[ P(t) = P_0 e^{kt} \]

\[ P(0) = P_0 \]

Find \( t \) when \( P(t) = 2P_0 \)

\[ 2P_0 = P_0 e^{kt} \]

\[ 2 = e^{kt} \]

\[ \ln 2 = kt \]

\[ \ln 2 = kt \Rightarrow t = \frac{\ln 2}{k} \approx \frac{0.69}{k} \]
\[
\begin{array}{c|c}
 t & z = f(t) \\
\hline
1 & 1 \\
10 & 100 \\
100 & 10,000
\end{array}
\Rightarrow
\begin{array}{c|c}
 x = \log t & y = \log z \\
\hline
0 & 0 \\
1 & 2 \\
2 & 4
\end{array}
\]

\[\log_t = \log_{10} t\]

\[\log(1) = 0 \quad \text{since } 10^0 = 1\]

\[\log(10) = 1 \quad \text{since } 10^1 = 10\]

\[\log(100) = 2 \quad \text{since } 10^2 = 100\]

\[y = 2x \Rightarrow z = t^2\]
Log-log plots

\[ z = f(t) \]

determine \( f \)

Let \( x = \log t \); \( y = \log z \)

Suppose \( y = mx + b \)

\( \log z = m \log t + b \)
\[
\log z = m \log t + b
\]

\[
10^\log z = 10^{m \log t + b}
\]

\[
z = (10^{m \log t})(10^b)
\]

\[
z = (10^b) t^m
\]

\[
z = A t^m
\]

A = 10^b

m = slope of line

y = mx + b

y-intercept
\[ y = 2x \]
\[ z = t^2 \]