\[ x^2 + y^2 = z^2 \] is constant

Is \( \frac{dx}{dt} \) increasing?

Yes \( (\to \infty) \)

\[ \frac{dx}{dt} = \frac{x}{\cos \theta} \frac{dz}{dt} \]

\[ \theta \to 90^\circ = \frac{\pi}{2} \Rightarrow \cos \theta \Rightarrow 0 \]
\[
\left( \frac{\sin x}{e^x} \right)' = \frac{e^x \cos x - (\sin x) e^x}{e^{2x}}
\]
\[
= \frac{\cos x - \sin x}{e^x}
\]

\[
y = \frac{\sin x}{e^x}
\]

\[
\ln y = \ln \left( \frac{\sin x}{e^x} \right)
\]
\[
= \ln(\sin x) - \ln(e^x)
\]
\[
\ln y = \ln(\sin x) - x
\]
\[
\frac{y'}{y} = \frac{\cos x}{\sin x} - 1
\]

\[
y' = y \left( \frac{\cos x}{\sin x} - 1 \right)
\]

\[
= \frac{\sin x}{e^x} \left( \frac{\cos x}{\sin x} - 1 \right)
\]

\[
= \frac{\cos x - \sin x}{e^x}
\]

\[
(e^x)' = e^x
\]

\[
(\ln x) = \frac{1}{x}
\]
4.3 Solve differential equation

Solve $y' = y$

Method 1: educated guessing

Let $y = e^x$

Check: $y' = e^x$

$y = e^x = y'$ ✓

Let $y = c e^x$

$y' = c e^x$

$\Rightarrow y = y'$

Soln: $y = c e^x$
Ex: Solve \( y' = ky \)

Guess \( y = Ce^{kx} \)

Check: \( y' = Ce^{kx} \cdot k \)

\[
\frac{dy}{dx} = ky
\]

\[
Ce^{kx} = k(Ce^{kx}) \checkmark
\]

\[
Ce^{kx} = \frac{1}{2}Ce^{kx} \checkmark
\]
Method 2: Solve \( y' = k y \)

\[
\frac{y'}{y} = k
\]

\( \ln y = kx + C_1 \)

\( e^{\ln y} = e^{kx+C_1} = e^{kx} e^{C_1} \)

\( y = C e^{kx} \)

Let \( C = e^{C_1} \)
Thm 8: Suppose $c$, $K$ constants

$$\frac{dy}{dx} = ky \iff y = ce^{kx}$$

Family of solutions depending on $c$

The rate of change of $y$ with respect to $x$ is proportional to $y$.
\[ y = \frac{1}{2} e^{cx} \]
\[ y' = \frac{1}{2} ce^{cx} \]
\[ y' = ky \]
\[ y' = \frac{1}{2} ce^{cx} \neq ky \]

Better guess \( y = Ce^{hx} \)
If rate of change of $y$ wrt $x$ is proportional to $y$

$y' = ky \iff y = ce^{kx}$

$k > 0$
Ex: Rate of growth of money is a CD

Ex: Rate of growth of bacteria in a closed petri dish under conditions of uninhibited growth

\[ \frac{dy}{dx} = ky \quad \longleftrightarrow \quad y = ce^{kx} \]

\[ F(x, t) > 0 \]

\[ y = 2e^{kx}, \quad c = 2 \]

\[ y = e^{kx}, \quad c = 1 \]

\[ y = \frac{1}{2}e^{kx}, \quad c = \frac{1}{2} \]

\[ y = -e^{kx}, \quad c = -1 \]
To specify which \( y = ce^{kx} \)

Need to choose a point \((x_0, y_0)\)

Initial value problem

Solve \( \frac{dy}{dx} = xy \) where \( y(x_0) = y_0 \) (i.e. \((x_0, y_0)\) lies on graph of the sol'n)

There exists a unique sol'n to this I.V.P.