By the chain rule $[(x^2+1)^9]' = 9(x^2+1)^8(2x)$

Or in other words, $\frac{d[(x^2+1)^9]}{dx} = 9(x^2+1)^8(2x)$

Or in other words, if we let $u = x^2+1$, then

$$\frac{du}{dx} = u' = 2x \text{ and}$$

$$[(x^2+1)^9]' = [u^9]' = 9u^8u' = 9(x^2+1)^8(2x)$$

Or in other notation,

$$\frac{d[(x^2+1)^9]}{dx} = \frac{d[u^9]}{dx} = 9u^8\frac{du}{dx} = 9(x^2+1)^8(2x)$$

We can use the chain rule to calculate a derivative using implicit differentiation.

Ex: Find the slope of the tangent line to $x^2+y^2=1$ at the point $(x,y) = (\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}})$.

Long method (without using implicit differentiation):

Solve for y:

$x^2+y^2=1$ implies $y^2 = 1-x^2$ implies $y = \pm\sqrt{1-x^2}$

Since the $y-$value of the point $(x,y) = (\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}})$ is negative, we are interested in the bottom half of the circle:

$$y = -\sqrt{1-x^2} = -(1-x^2)^{\frac{1}{2}}$$

To find slope of the tangent line, take derivative:

$$\frac{dy}{dx} = -\frac{1}{2}(1 - x^2)^{\frac{-1}{2}}(-2x) = \frac{x}{\sqrt{1-x^2}}$$

Hence when $x = \frac{1}{\sqrt{2}}$, then the slope of the tangent line is

$$\frac{\frac{1}{\sqrt{2}}}{\sqrt{1-(\frac{1}{\sqrt{2}})^2}} = \frac{\frac{1}{\sqrt{2}}}{\sqrt{1-(\frac{1}{2})}} = \frac{\frac{1}{\sqrt{2}}}{\sqrt{\frac{1}{2}}} = \frac{\sqrt{\frac{1}{2}}}{\sqrt{\frac{1}{2}}} = 1$$

We can instead use implicit differentiation:

Note that y is a function of x for the bottom half of the circle: $y = f(x) = -(1 - x^2)^{\frac{1}{2}}$

Thus to find the derivative of y^2, we can use the chain rule:

$$\frac{d(y^2)}{dx} = 2y \cdot \frac{dy}{dx} = 2(-(1 - x^2)^{\frac{1}{2}}) \cdot \frac{x}{\sqrt{1-x^2}} = -2x$$

Note $y^2 = [-(1 - x^2)^{\frac{1}{2}}]^2 = 1 - x^2$

However, we don't need the above to find the slope of the tangent line to the unit circle at the point $(x, y) = (\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}})$. Instead:

Shorter method for finding this slope of the tangent line to $x^2 + y^2 = 1$ at the point $(x, y) = (\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}})$.

We have $x^2 + y^2 = 1$, and we want to find slope $= \frac{dy}{dx}$

Take the derivative with respect to x of both sides:

$$\frac{d(x^2 + y^2)}{dx} = \frac{d(1)}{x}$$

$$2x + 2y \cdot \frac{dy}{dx} = 0$$

Solve for $\frac{dy}{dx}$: $\qquad 2y \cdot \frac{dy}{dx} = -2x$

$$\frac{dy}{dx} = \frac{-x}{y}$$

Hence the slope of the tangent line to the unit circle at the point $(x, y) = (\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}})$.

$$\frac{dy}{dx} = \frac{-x}{y} = \frac{-\frac{1}{\sqrt{2}}}{-\frac{1}{\sqrt{2}}} = 1$$

Suppose $2x^2y - 3y^2 = 4$. First find y':

Easiest method is to use implicit differentiation. Take derivative (with respect to x) of both sides.

$$\frac{d}{dx}(2x^2y - 3y^2) = \frac{d}{dx}(4)$$

$$4xy + 2x^2y' - 6yy' = 0$$

Solve for y' (note this step is easy as one can factor y' from some terms. Observe that this will always be the case):

$$y'(2x^2 - 6y) = -4xy$$

$$y' = \frac{-4xy}{2x^2-6y} = \frac{-2(2xy)}{-2(3y-x^2)} = \frac{2xy}{3y-x^2}$$

Hence $y' = \frac{2xy}{3y-x^2}$