Chapter 2 Basic Counting

2.1 Product Rule: If \( S = S_1 \times S_2 \), then \(|S| = |S_1||S_2|\).

\( x = (a, b) \in S \) implies \( a \in S_1 \) AND \( b \in S_2 \), then \(|S| = |S_1||S_2|\).

How many sequences consisting of one letter followed by one single digit number (0 - 9) are possible?

How many different license plates are possible if 3 letters followed by 3 numbers are used?

How many different DNA sequences of length 2?

How many different DNA sequences of length 3?

2.6 Subsets

Suppose a set \( A \) has four elements (i.e., the cardinality of \( A = |A| = 4 \))

The number of subsets of \( A \) is

The number of nonempty subsets of \( A \) is

Suppose we know proteins \( A, B, C, D \) affect a particular biological reaction. How many different experiments can be performed in order to analyze the effects of these proteins on the biological reaction?

A pizza parlor offers 4 different toppings (sausage, onions, chicken, walnuts). How many different types of pizzas can one order?

Suppose a set \( B \) has \( n \) elements (i.e., \( |B| = n \)). The number of subsets of \( B \) is
2.2 Sum Rule: If $S = S_1 \cup S_2$ and $S_1 \cap S_2 = \emptyset$, then $|S| = |S_1| + |S_2|$.

If $S_1 \cap S_2 = \emptyset$ and if $x \in S$ implies $x \in S_1$ OR $x \in S_2$, then $|S| = |S_1| + |S_2|$.

Suppose a symbol can be either a number between 0 and 9 or a letter. How many are symbols there?

How many even numbers between 100 and 1000 have distinct digits.

2.3, 2.5 Permutations and $r$-permutations:

Suppose $|S| = n$.

An $r$-permutation of $S$ is an ordered arrangement of $r$ of the $n$ elements of $S$.

If $r = n$, then an $r$-permutation of $S$ is a permutation of $S$.

$P(n,r) =$ number of $r$-permutations of $S$ where $|S| = n$.

4 TA’s need to be assigned to 4 different classes. How many different possible assignments are there?

4 classes need to be assigned a TA. There are 10 TAs. How many different possible assignments are there?

Defn: $n! = n(n-1)(n-2)...(2)(1)$, $0! = 1$

Thm 3.2.1: If $r > n$, then $P(n,r) = 0$.
If $r \leq n$, then $P(n,r) = \frac{n!}{(n-r)!}$

$P(0,0) = \quad P(n,0) = \quad P(n,1) = \quad P(n,n) =$
2.7 r-Combinations

An r-combination of S is an r-element subset of S (ORDER DOES NOT MATTER).

\[ C(n, r) = \text{number of r-combinations of } S \text{ where } |S| = n. \]

How many different math teams consisting of 4 people can be formed if there are 10 students from which to choose?

Thm: \[ C(n, r) = \binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{P(n,r)}{r!} \]

Cor: \[ C(n, r) = C(n, n - r) \]

Cor: \[ C(n, r) = C(n - 1, r - 1) + C(n - 1, r) \]

Cor: Pascal’s Triangle.

Cor: \[ \sum_{i=0}^{n} \binom{n}{i} = 2^n \]

How many different proteins containing 10 amino acids can be formed if the protein contains 5 alanines (A), 3 leucines (L), and 2 serines (S)?