Review problems (note these problems do NOT cover everything thing that may be on the final).

1.) Let $X = \{1, 2\} \times \{1, 2\}$ Define the relation F on X by $(a, b)R(c, d)$ iff $ad = bc$. Draw R as a subset of $X \times X$. Determine which of the following properties hold for R (Prove it).

Is R reflexive?

Is R irreflexive?

Is R symmetric?

Is R antisymmetric?

Is R transitive?

Is R an equivalence relation?

If so, use R to partition X into its equivalence classes.

Is R a partial order?

Give an example of a partial order on X.

2.) Find the number of necklaces you can create containing 4 beads if you have exactly 20 beads and each bead is unique?

3.) Find the number of necklaces you can create containing 4 beads if you have exactly 1 red bead, 1 blue bead, and two identical green beads.

4.) Find the number of necklaces you can create containing 21 beads if the necklace must contain 1 red bead and any number of yellow beads or blue beads.

Hint: Place red bead first and then use Burnside's lemma (as described in answers).

5.) Suppose a code is created using the numbers $\{0, 1, 2\}$. Suppose the code must contain an even number of 0's. How many different codes of length n can be created?

Hint: determine the linear non-homogeneous recurrence relation $h_n = h_{n-1} + 3^{n-1}$, $h_0 = 1$ by breaking the counting into two cases: $x_n \neq 0$ and $x_n = 0$. Then solve the the linear non-homogeneous recurrence relation. Note: $h_n = a(3^n)$ is a solution to the non-homogeneous recurrence relation for some value of a.

6a.) Suppose someone claims that they used their computer for a total of 241 hours over a 10 day period. Do you believe this person?

6b.) Suppose this person lives on a different planet. Suppose the time it takes for this planet to rotate 360 degrees around its axis corresponds to an integral number of hours as measured in earth time. Suppose also that each day the person uses their computer for an integral number of hours for a total of 241 hours over a 10 day period. What is the minimum possible time it takes for this planet to rotate 360 degrees around its axis.

7.) Suppose S is a collections of 6 integers. Show that there exists $x, y \in S$ such that $x \neq y$, but $x - y$ is a multiple of 5.

8.) In how many ways can 12 indistinguishable apples and 2 oranges be distributed among three children in such a way that each child gets at least 2 pieces of fruit?

9.) Use combinatorial reasoning to prove that $\sum_{k=0}^{n} k(k-1)\binom{n}{k} = n(n-1)2^{n-2}$.

10.) Suppose Ann, Beth, Carl, Don, and Edna are to be assigned jobs 1, 2, 3, 4, 5. Suppose Ann is qualified for jobs 1, 2, 3; Beth is qualified for jobs 4, 5; Carl is qualified for jobs 1, 4, 5; Don is qualified for jobs 1, 2, 3; and Edna is qualified for jobs 1, 2, 4, 5. How many different job assignments are possible if each person is assigned exactly one job for which they are qualified?

Hint: create a chessboard where columns correspond to Ann, Beth, Carl, Don, and Edna and rows correspond to jobs forbidden to them. Place Ann and Carl in the last two columns so that you can partition the chessboard into two sets A (containing 6 forbidden positions) and B (containing 4 forbidden positions).