n-Permutation of n objects: \( P(n, n) = n! = \) number of ways to place n nonattacking rooks on an \( n \times n \) chessboard.

r-Permutation of n objects: \( P(n, r) = n(n-1)...(n-r+1) = \) number of ways to place r nonattacking rooks on an \( r \times n \) chessboard, \( r \leq n \).

\[ C(n, r)P(n, r) = \frac{n(n-1)...(n-r+1)}{r!} = \text{number of ways to place } r \text{ nonattacking rooks on an } n \times n \text{ chessboard, } r \leq n. \]

### 2.4 Permutations of Multisets

Thm 2.4.1: Let \( A = \{\infty \cdot 1, \infty \cdot 2, \ldots, \infty \cdot k\} \)

The number of \( r \) permutations of \( A = k^r \).

Thm 2.4.2: Let \( B = \{n_1 \cdot 1, n_2 \cdot 2, \ldots, n_k \cdot k\} \)

\( n = n_1 + n_2 + \ldots + n_k \)

The number of \( n \)-permutations of \( B = \frac{n!}{n_1!n_2!\ldots n_k!} \).

If want \( r \)-permutations of \( B \), need to use or statements or technique from later chapter.

Combinations: order does NOT matter

\[ \binom{n}{r} = \# \text{ of } r \text{ combinations of } \{1, 2, \ldots, n\} \]

\[ = \text{subsets of } \{1, 2, \ldots, n\} \text{ containing exactly } r \text{ elements.} \]

\[ 2^n = \sum_{i=0}^{n} \binom{n}{i} = \# \text{ of subsets of } \{1, 2, \ldots, n\}. \]

Pascal’s Triangle: \( C(n, r) = C(n-1, r-1) + C(n-1, r) \)

### 2.4 Combinations of Multisets

Let \( A = \{\infty \cdot 1, \infty \cdot 2, \ldots, \infty \cdot k\} \)

The number of \( r \) combinations of \( A = \binom{r+k-1}{r} \)

\[ = \# \text{ of solutions to } x_1 + x_2 + \ldots + x_k = r \text{ such that } x_i \geq 0, x_i \in \mathbb{Z} \]

\[ = \# \text{ of permutations of } \{r \cdot 1, (k-1) \cdot +\} \]

\[ = \text{partitions of } r \text{ indistinguishable objects into } k \text{ distinguishable boxes.} \]

If need \( x_i \geq a_i \) replace \( x_i \) with \( y_i + a_i \) since \( y_i = x_i - a_i \geq 0 \)