

$n$ -Permutation of  $n$  objects:  $P(n, n) = n! =$  number of ways to place  $n$  nonattacking rooks on an  $n \times n$  chessboard.

$r$ -Permutation of  $n$  objects:  $P(n, r) = n(n-1)\dots(n-r+1) =$  number of ways to place  $r$  nonattacking rooks on an  $r \times n$  chessboard,  $r \leq n$ .

$C(n, r)P(n, r) = \frac{[n(n-1)\dots(n-r+1)]^2}{r!} =$  number of ways to place  $r$  nonattacking rooks on an  $n \times n$  chessboard,  $r \leq n$ .

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#### 2.4 Permutations of Multisets

Thm 2.4.1: Let  $A = \{\infty \cdot 1, \infty \cdot 2, \dots, \infty \cdot k\}$

The number of  $r$  permutations of  $A = k^r$ .

Thm 2.4.2: Let  $B = \{n_1 \cdot 1, n_2 \cdot 2, \dots, n_k \cdot k\}$

$$n = n_1 + n_2 + \dots + n_k$$

The number of  $n$ -permutations of  $B = \frac{n!}{n_1!n_2!\dots n_k!}$ .

If want  $r$ -permutations of  $B$ , need to use or statements or technique from later chapter.

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Combinations: order does NOT matter

$$\binom{n}{r} = \# \text{ of } r \text{ combinations of } \{1, 2, \dots, n\}$$

= subsets of  $\{1, 2, \dots, n\}$  containing exactly  $r$  elements.

$$2^n = \sum_{i=0}^n \binom{n}{i} = \# \text{ of subsets of } \{1, 2, \dots, n\}.$$

Pascal's Triangle:  $C(n, r) = C(n-1, r-1) + C(n-1, r)$

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#### 2.4 Combinations of Multisets

Let  $A = \{\infty \cdot 1, \infty \cdot 2, \dots, \infty \cdot k\}$

The number of  $r$  combinations of  $A = \binom{r+k-1}{r}$

= # of solutions to  $x_1 + x_2 + \dots + x_k = r$  such that  $x_i \geq 0, x_i \in \mathcal{Z}$

= # of permutations of  $\{r \cdot 1, (k-1) \cdot +\}$

= partitions of  $r$  indistinguishable objects into  $k$  distinguishable boxes.

if need  $x_i \geq a_i$  replace  $x_i$  with  $y_i + a_i$  since  $y_i = x_i - a_i \geq 0$