7.4: Generating Functions

$g(x) = h_0 + h_1 x + h_2 x^2 + ...$ is the *generating function* for the sequence $h_0, h_1, h_2, ...$

Ex: The generating fn for the sequence 2, 3, 4, 0, 0, 0, ... is

$g(x) = 2 + 3x + 4x^2$

Ex: The generating function for the sequence 1, 1, 1, ... is

$g(x) = 1 + x + x^2 + ... = \frac{1}{1-x}$

Ex: The generating function for the sequence

$\binom{m}{0}, \binom{m}{1}, \binom{m}{2}, ..., \binom{m}{m}$ is

$$g(x) = \binom{m}{0} + \binom{m}{1} x + \binom{m}{2} x^2 + ... \binom{m}{m} x^m = (1+x)^m$$

Ex: Suppose $\alpha \in \mathcal{R}$. The generating function for the sequence

$\binom{\alpha}{0}, \binom{\alpha}{1}, \binom{\alpha}{2}, ...,$ is

$$g(x) = \binom{\alpha}{0} + \binom{\alpha}{1} x + \binom{\alpha}{2} x^2 + ... = (1+x)^\alpha$$

Ex: Let h_n = number of nonnegative solutions to $e_1 + e_2 + ... + e_k = n$

Thus $h_n =$

Thus $g(x) =$

Suppose a multiset consisting of integers between 0 and 5 inclusive of size k must contain the following:

even number of 0's

odd number of 1's

three or four 2's

the number of 3's is a multiple of five

between zero to four (inclusive) 4's

zero or one 5

Find the number of multisets of size k.

Find the number of multisets of size 100.