Math 150 Exam 2 Form C
October 28, 2015

[8] 1.) In the expansion of $(2x + 5y - z - 1)^{10}$,

the coefficient of $x^4y^3z^5$ is ____________________

the coefficient of x^2yz is ____________________

[13] 2.) Let $S = \{x_6, x_5, x_4, x_3, x_2, x_1, x_0\}$.

What subset of S corresponds to 1101101? ______________________________

What subset comes before the subset $\{x_4\}$? ______________________________

What subset comes after the subset $\{x_4\}$? ______________________________

[10] 3.) How many permutations of $\{1, 2, 3, 4, 5, 6, 7\}$

a.) have exactly 20 inversions? _______________

a.) have exactly 21 inversions? _______________

a.) have exactly 22 inversions? _______________

[9] 4.) Draw the Hasse Diagram for the inversion poset $(X_3, \leq)$ where $X_3 =$ the set of permutations of $\{1, 2, 3\}$ and if π and σ are two permutations in X_3, then $\pi \leq \sigma$ if the set of inversions of π is a subset of the set of inversions of σ.

2pts Extra credit: Prove that $r(3, 3) \geq 6$ (Note this problem is not the same as 6A).

[20] 5.) State the definition of equivalence relation:

Use the definition of equivalence relation to show the $\sim$ is an equivalence relation on $\mathbb{Z}$ where $n \sim k$ iff $\frac{n-k}{4} \in \mathbb{Z}$

What are the equivalence classes of $\mathbb{Z}$ with respect to $\sim$?

Partition $\mathbb{Z}$ into its equivalence classes (I.e., write $\mathbb{Z}$ as the disjoint union of sets where the sets correspond to equivalence classes.

[40] 6.) Choose 2 from the following 3 problems. **Circle your choices: A B C** You may do all 3 problems in which case your unchosen problem can replace your lowest scoring problem at 4/5 the value (or more) as discussed in class.

Note: If you do not CLEARLY indicate your 2 choices, I will assume that you chose the first two problems.

6A.) Prove that $r(3,3) \leq 6$

6B.) Use a combinatorial argument to prove the Vandermonde convolution for the binomial coefficients: For all positve integers m_1, m_2, n,

$$\sum_{k=0}^{n} \binom{m_1}{k} \binom{m_2}{n-k} = \binom{m_1 + m_2}{n}$$

6C.) State Newton's binomial theorem for expanding $(x+y)^\alpha$ where $\alpha \in \mathbb{R}$.

Use this theorem to algebraically derive the formula: $\frac{1}{1-z} = \sum_{k=0}^{\infty} z^k$ when $|z| < 1$.

Hint: Let $\alpha = -1$. You may use the fact that $\binom{-n}{k} = (-1)^k \binom{n+k-1}{k}$