Generate all subsets of $\{x_{n-1}, x_{n-2}, \dots, x_1, x_0\}$

To generate combinations (subsets), generate binary numbers.

Ex: $\{x_2, x_1, x_0\}$

#	binary number	→	subset
0	000	→	$\emptyset$
1	001	→	$\{x_0\}$
2	010	→	$\{x_1\}$
3	011	→	$\{x_1, x_0\}$
4	100	→	$\{x_2\}$
5	101	→	$\{x_2, x_0\}$
6	110	→	$\{x_2, x_1\}$
7	111	→	$\{x_2, x_1, x_0\}$

Order the binary numbers/subsets using lexigographical order = dictionary order.

Note that we start counting from 0.

Thus the 0th subset in our list corresponds to the binary number 000 = $\emptyset$. The first subset in our list coresponds to the binary number 001 = $\{x_0\}$

Powers of 2

0	1	2	3	4	5	6	7	8
2^0	2^1	2^2	2^3	2^4	2^5	2^6	2^7	2^8
1	2	4	8	16	32	64	128	256

Find subset #0 of $\{x_7, x_6, x_5, x_4, x_3, x_2, x_1, x_0\}$: $\emptyset$

0 0 0 0 0 0 0 0

Find subset #1 of $\{x_7, x_6, x_5, x_4, x_3, x_2, x_1, x_0\}$: $\{x_0\}$

0 0 0 0 0 0 0 1

Find subset #15 of $\{x_7, x_6, x_5, x_4, x_3, x_2, x_1, x_0\}$

15= $2^3 + 2^2 + 2^1 + 2^0 \longleftrightarrow 00001111$

$\boxed{\{x_3, x_2, x_1, x_0\}}$

Find subset #16 of $\{x_7, x_6, x_5, x_4, x_3, x_2, x_1, x_0\}$

16 = $2^4 \longrightarrow 00010000$

$\boxed{\{x_4\}}$

Find subset #37 of $\{x_7, x_6, x_5, x_4, x_3, x_2, x_1, x_0\}$

37 = $2^5 + 5 = 2^5 + 2^2 + 2^0 \longrightarrow 00100101$

$\boxed{\{x_5, x_2, x_0\}}$

1001111

What comes after $\{x_6, x_3, x_2, x_1, x_0\}$?

1001111
$+ \quad\quad 1$
1010000

$\boxed{\{X_6, X_4\}}$

Find the 15th combination of $\{x_5, x_4, x_3, x_2, x_1, x_0\}$

$\{X_3, X_2, X_1, X_0\}$ ⇐ see page 1

4.5: R is a *relation on a set* X if $R \subset X \times X$.

aRb if $(a, b) \in R$. $\quad\quad a\not{R}b$ if $(a, b) \notin R$.

R is *reflexive* if $xRx \ \forall x \in X$.

diagonal included

R is *irreflexive* if $x\not{R}x \ \forall x \in X$.

EX: $<$, $\subsetneq$

R is *symmetric* if xRy implies yRx. $(k, n) \in \cong_p \Leftrightarrow (n, k) \in \cong_p$

$\cong_p \quad\quad \frac{k-n}{p} \in \mathbb{Z}$ then $\frac{n-k}{p} \in \mathbb{Z}$

R is *antisymmetric* if xRy and yRx implies $x = y$.

$\leq \quad\quad$ if $x \leq y$ & $y \leq x \Rightarrow x = x$

R is *transitive* if xRy and yRz, then xRz.

$\leq \quad\quad x \leq y$ & $y \leq z \Rightarrow x \leq z$

Defn: A *partial order* ($\leq$)is
reflexive, anti-symmetric, and transitive.

Defn: A *strict partial order* ($<$)is
irreflexive, anti-symmetric, and transitive.

Note: If $\leq \ \subset \ X \times X$ is a partial order, then
$< \ = \ \leq$ - the diagonal is a strict partial order.

Defn: x and y are *comparable* if xRy or yRx.
Else x and y are *incomparable*.

Defn: A *total order* is a partial order where every pair of elements of X are comparable.

Defn: An *equivalence relation* is
reflexive, symmetric, and transitive.