

$f : A \rightarrow B$ is 1:1 iff $f(x_1) = f(x_2)$ implies $x_1 = x_2$.

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Hypothesis: $f(x_1) = f(x_2)$. Conclusion $x_1 = x_2$.

Hypothesis implies conclusion.

p implies q .

$p \Rightarrow q$.

Note a statement, $p \Rightarrow q$, is true if whenever the hypothesis p holds, then the conclusion q also holds.

To prove that a statement is true:

- (1) Assume the hypothesis holds.
- (2) Prove the conclusion holds.

Ex: To prove a function is 1:1: arbitrary hyp holds
(1) Assume $f(x_1) = f(x_2)$
(2) Do some algebra to prove $x_1 = x_2$.

$[p \Rightarrow q]$ is equivalent to $\forall p, q$ holds.

That is, for everything satisfying the hypothesis p , the conclusion q must hold.

Compare to HW3
handout

A statement is false if the hypothesis holds, but the conclusion need not hold.

Hypothesis does not implies conclusion.

p does not imply q .

$p \not\Rightarrow q$.

That is there exists a **specific case** where the hypothesis holds, but the conclusion does not hold.

To prove that a statement is false:

Find an example where the hypothesis holds, but the conclusion does not hold.

Ex: To prove a function is not 1:1, find specific x_1, x_2 such that $f(x_1) = f(x_2)$, but $x_1 \neq x_2$. *satisfy p*

Ex: $f : R \rightarrow R, f(x) = x^2$ is not 1:1 since $f(1) = 1^2 = 1 = (-1)^2 = f(-1)$, but $1 \neq -1$

$\sim [p \Rightarrow q]$ is equivalent to $\sim [\forall p, q \text{ holds}]$.

Thus if $p \Rightarrow q$ is false, then it is not true that $[\forall p, q \text{ holds}]$. That is, $\exists p$ such that q does not hold.

$p \Rightarrow q$ is false

$p \Rightarrow q$ is true, then iff its contrapositive $\sim q \Rightarrow \sim p$ is also true.

equivalent

But its converse, $q \Rightarrow p$ may not be true.

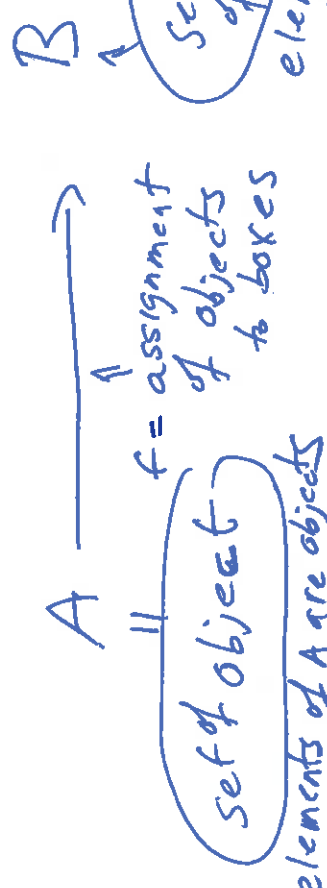
inverses

$\sim p \Rightarrow \sim q$

Thm 3.1.1: Pigeonhole Principle (weak form): If you have $n + 1$ objects placed in n boxes, then at least one box will be occupied by 2 or more objects.

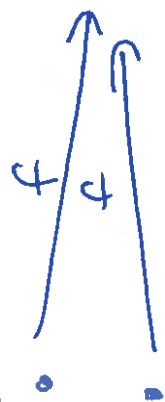
Thm 3.1.1: Pigeonhole Principle (weak form): If you have $n + 1$ pigeons in n pigeonholes, then at least one pigeonhole will be occupied by 2 or more pigeons.

objects boxes

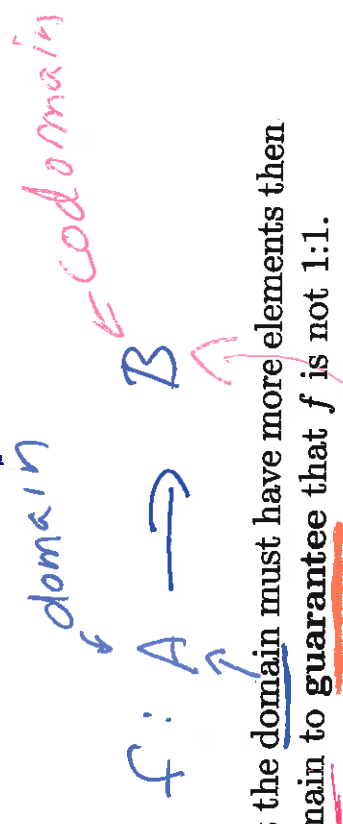


Thm 3.1.1: If $f : A \rightarrow B$ is a function and $|A| = n + 1$, and $|B| = n$, then f is not 1:1.

Cor: If $f : A \rightarrow B$ is a function and A is finite and $|A| > |B|$, then f is not 1:1.



not 1:1



Note that the domain must have more elements than the codomain to guarantee that f is not 1:1.

Recall the converse of $[p \text{ implies } q]$ is $[q \text{ implies } p]$.

Note the converse of a theorem is frequently false as the following example illustrates:

$c : \{1, \dots, n\} \rightarrow \{1, \dots, n\}, c(k) = 1$ is not 1:1, a constant fn

but domain does not have more elements than the codomain. Thus converse is false

$f : A \rightarrow B$ a function which is not 1:1 does not imply $|A| > |B|$.

Contrapositive of $[p \text{ implies } q]$ is $[\sim q \text{ implies } \sim p]$. The contrapositive of a theorem is true:

Cor: If $f : A \rightarrow B$ is a function which is 1:1, then $|A| \leq |B|$.

Related theorem:

Thm: If $f : A \rightarrow B$ is a function and if $|A| = n = |B|$, then f is 1:1 iff f is onto.