$f : A \to B$ is 1:1 iff $f(x_1) = f(x_2)$ implies $x_1 = x_2$.

$$f(x_1) = f(x_2) \text{ implies } x_1 = x_2.$$

Hypothesis: $f(x_1) = f(x_2)$.   Conclusion $x_1 = x_2$.

Hypothesis implies conclusion.
p implies q.
$$p \Rightarrow q.$$

Note a statement, $p \Rightarrow q$, is true if whenever the hypothesis p holds, then the conclusion q also holds.

To prove that a statement is true:
(1) Assume the hypothesis holds.
(2) Prove the conclusion holds.

arbitrary x_1 & x_2 st hyp holds

Ex: To prove a function is 1:1:
(1) Assume $f(x_1) = f(x_2)$
(2) Do some algebra to prove $x_1 = x_2$.

$[p \Rightarrow q]$ is equivalent to $[\forall p,\ q \text{ holds}]$.

That is, for everything satisfying the hypothesis p, the conclusion q must hold.

Compare to HW 3 handout

A statement is false if the hypothesis holds, but the conclusion need not hold.

Hypothesis does not implies conclusion.
p does not imply q.
$$p \not\Rightarrow q.$$

That is there exists **a specific case** where the hypothesis holds, but the conclusion does not hold.

To prove that a statement is false:

Find an example where the hypothesis holds, but the conlusion does not hold.

$\exists\, x_1, x_2$ satisfy p

Ex: To prove a function is not 1:1, find specific x_1, x_2 such that $f(x_1) = f(x_2)$, but $x_1 \neq x_2$.

Ex: $f : R \to R$, $f(x) = x^2$ is not 1:1
since $f(1) = 1^2 = 1 = (-1)^2 = f(-1)$, but $1 \neq -1$

$\sim [p \Rightarrow q]$ is equivalent to $\sim [\forall p,\ q \text{ holds}]$.
not   not

Thus if $p \Rightarrow q$ is false,
then it is not true that $[\forall p,\ q \text{ holds}]$.
That is, $\exists p$ such that q does not hold.

$p \Rightarrow q$ is false

~~If~~ $p \Rightarrow q$ is true, ~~then~~ iff

its contrapostive $\sim q \Rightarrow \sim p$ is also true.

equivalent

But its converse, $q \Rightarrow p$ may not be true.

inverse $\sim p \Rightarrow \sim q$

Thm 2.1.1: Pigeonhole Principle (weak form): If you have $n+1$ objects placed in n boxes, then at least one box will be occupied by 2 or more objects.

Thm 2.1.1: Pigeonhole Principle (weak form): If you have $n+1$ pigeons in n pigeonholes, then at least one pigeonhole will be occupied by 2 or more pigeons.

objects   boxes

$A \longrightarrow B$

A = set of object

f = assignment of objects to boxes

B = set of Boxes

elements of A are objects

elements of B are boxes

Thm 2.1.1: If $f : A \to B$ is a function and $|A| = n+1$, and $|B| = n$, then f is not 1:1.

Cor: If $f : A \to B$ is a function and A is finite and $|A| > |B|$, then f is not 1:1.

f f not 1:1

$f : A \longrightarrow B$

domain   codomain

Note that the domain must have more elements then the codomain to **guarantee** that f is not 1:1.

Recall the *converse* of [p implies q] is [q implies p].

Note the converse of a theorem is frequently false as the following example illustrates:

$$c : \{1, ..., n\} \to \{1, ..., n\},\ c(k) = 1 \text{ is not } 1:1,$$

a constant fn

but domain does not have more elements than the codomain. Thus converse is false

$f : A \to B$ a function which is not 1:1 does not imply $|A| > |B|$.

Contrapositive of [p implies q] is [$\sim q$ implies $\sim p$]. The contrapositive of a theorem is true:

Cor: If $f : A \to B$ is a function which is 1:1, then $|A| \leq |B|$.

Related theorem:
Thm: If $f : A \to B$ is a function and if $|A| = n = |B|$, then f is 1:1 iff f is onto.