$f: A \to B$ is $1:1$ (injective) iff

$$\underbrace{f(x_1) = f(x_2)}_{\text{Hypothesis}} \Rightarrow \underbrace{x_1 = x_2}_{\text{conclusion}}$$

Hypothesis $\Rightarrow$ conclusion

$$p \Rightarrow q$$

Note: A statement is true if whenever the hypothesis holds, then the conclusion holds.

To prove a statement $(p \Rightarrow q)$ is true

1) Assume hypothesis
2) Prove Conclusion

EX: To prove fn $f: A \to B$ is $1:1$

1) $f(x_1) = f(x_2)$
2) Do some algebra to show $x_1 = x_2$ (ie solve for x_1)

A statement is false if hypothesis holds for some case

But the conclusion does not hold

A single exception makes statement false

To prove a fn is not 1:1

Find specific x_1 & x_2 st

$$f(x_1) = f(x_2)$$ ← hyp holds

but $x_1 \neq x_2$ ← concl does not hold

$f : A \to B$ is 1:1 iff $f(x_1) = f(x_2)$ implies $x_1 = x_2$. $\leftarrow$ $\boxed{p \Rightarrow q}$

$f : A \to B$ is 1:1 iff $x_1 \neq x_2$ implies $f(x_1) \neq f(x_2)$. $\leftarrow$ $\boxed{\sim q \Rightarrow \sim p}$

$f : A \to B$ is 1:1 iff for all $x_1 \neq x_2$, $f(x_1) \neq f(x_2)$.

$f : A \to B$ is NOT 1:1 iff there exists $x_1 \neq x_2$ such that $f(x_1) = f(x_2)$.

$1 \to 1$, $-1 \to 1$

Determine if the following functions are 1:1. Prove it.

1.) $f : R \to R$, $f(x) = x^2$

$f(1) = 1 = f(-1)$ since $(1)^2 = (-1)^2$, thus f is NOT 1:1

2.) $f : [0, \infty) \to R$, $f(x) = x^2$

Suppose $f(x_1) = f(x_2)$ (hypothesis) $\Rightarrow x_1^2 = x_2^2 \Rightarrow x_1 = \pm\sqrt{x_2^2} = \pm|x_2|$

but $x_1, x_2 \geq 0$

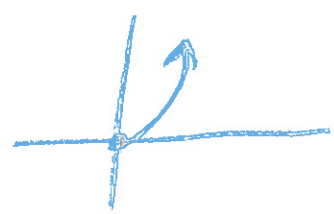
concl

$\Rightarrow \boxed{x_1 = x_2}$ $\Rightarrow$ f is 1:1

3.) $f : [0, \infty) \to [0, \infty)$, $f(x) = x^2$

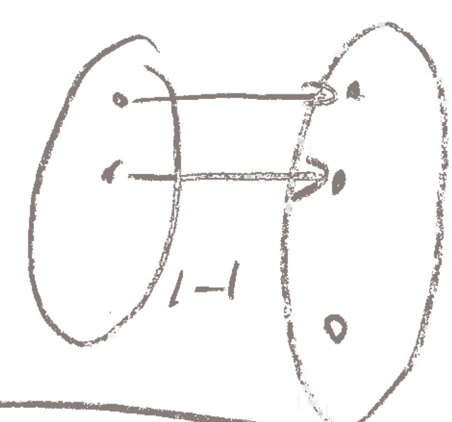
same as for #2 : 1:1

4.) $f : R \to R$, $f(x) = x^3$



5.) $f : R \to R$, $f(x) = 2$

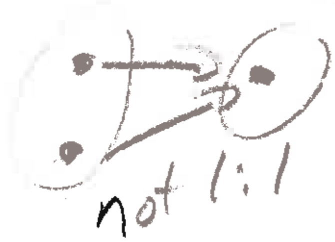
6.) $f : R \to R$, $f(x) = 8x + 2$



7.) $f : R \to R$, $f(x) = x^2 + 3x$

8.) $f : R \to R$, $f(x) = e^x$

9.) $f : R \to R$, $f(x) = x^4 + x^2$



10.) $f : R \to R$, $f(x) = sin(x)$

domain codomain image

$f: A \to B$ is a fn $\Rightarrow f(A) \subset B$

For onto $f(A) \supset B$

$B \subset f(A)$

$f : A \to B$ is onto iff $\underbrace{f(A)}_{\text{image}} = B$.

$f : A \to B$ is onto iff $b \in B$ implies there exists an $a \in A$ such that $f(a) = b$. $b \in f(A)$

$f : A \to B$ is onto iff for all $b \in B$, there exists an $a \in A$ such that $f(a) = b$.

$f : A \to \boxed{B}$ is NOT onto iff there exists $\boxed{b \in B}$ s. t. there does not exist an $a \in A$ s. t. $f(a) = b$.

Determine if the following functions are onto. If a function is not onto, prove it.

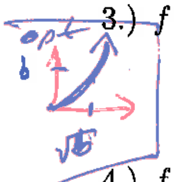
1.) $f : R \to \boxed{R}, f(x) = x^2$

$-1 \in \mathbb{R}$, but $f(x) = x^2 = -1$ has no sol'n so -1 is not in image of f $\Rightarrow f$ is NOT onto

2.) $f : [0, \infty) \to R, f(x) = x^2$

NOT ONTO, proof is same as for #1

3.) $f : [0, \infty) \to [0, \infty), f(x) = x^2$



onto

4.) $f : R \to R, f(x) = x^3$

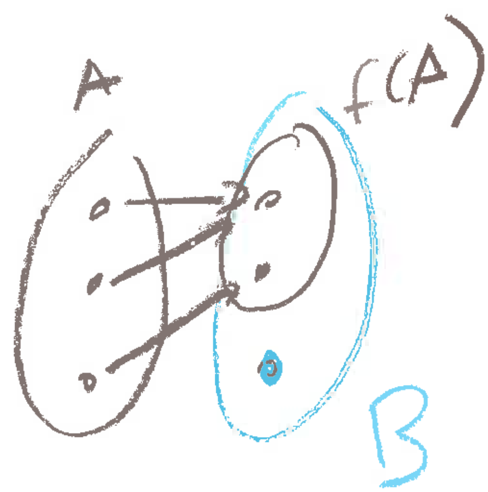
5.) $f : R \to R, f(x) = 2$

onto = surjective

6.) $f : R \to R, f(x) = 8x + 2$

7.) $f : R \to R, f(x) = x^2 + 3x$

8.) $f : R \to R, f(x) = e^x$



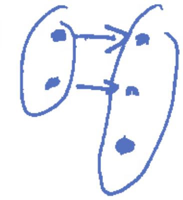
9.) $f : R \to R, f(x) = x^4 + x^2$

10.) $f : R \to R, f(x) = sin(x)$

Not onto

invertible

bij



not bij

1:1, but not 1:1 correspondence

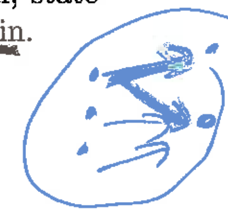
$|A| = |B|$ iff there exists a bijection $f : A \to B$.

$f : A \to B$ is a bijection iff f is 1:1 and f is onto.

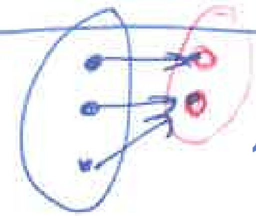
1-1 correspondence and onto 1-1

f is NOT a bijection iff f is not 1:1 OR f is not onto.

Determine if the following functions are bijections. If a function is not a bijection, state why and determine if you can create a bijective function by changing the co-domain.



not a function



not 1-1

1.) $f : R \to R$, $f(x) = x^2$

Not bijective

Not 1:1, not onto

2.) $f : [0, \infty) \to R$, $f(x) = x^2$

Not bijective since not onto

$g : [0,\infty) \to [0,\infty)$ $g(x) = x^2$ is bijective

3.) $f : [0, \infty) \to [0, \infty)$, $f(x) = x^2$

bijection

not onto but $f : A \to f(A)$ is onto

4.) $f : R \to R$, $f(x) = x^3$

5.) $f : R \to R$, $f(x) = 2$

6.) $f : R \to R$, $f(x) = 8x + 2$

7.) $f : R \to R$, $f(x) = x^2 + 3x$

8.) $f : R \to R$, $f(x) = e^x$

9.) $f : R \to R$, $f(x) = x^4 + x^2$

10.) $f : R \to R$, $f(x) = sin(x)$