2.6 Finite probability

Suppose $E \subset S$, then the probability of $E = P(E) = \frac{|E|}{|S|}$

S = sample space, E = events.

Note: we assume each outcome is equally likely

Ex: A football season consists of 11 games. What is the probability that the season ends in 7 wins, 2 losses, and 2 ties, IF it is equally likely that the football team wins, looses, or ties.

$P(\text{winning}) = 1/3 \quad P(\text{loosing}) = 1/3 \quad P(\text{tie}) = \frac{1}{3}$

The number of ways the season can end in 7 wins, 2 losses, and 2 ties is

# of permutations of $\{7 \cdot w, 2 \cdot \ell, 2 \cdot t\}$

$$= \frac{11!}{7!\,2!\,2!}$$

The number of different ways in which the season can end is

# of 11-permutations of $\{\infty \cdot w, \infty \cdot \ell, \infty \cdot t\}$

$\sim \{11 \cdot w, 11 \cdot \ell, 11 \cdot t\}$

$$= 3 \cdot 3 \cdot \ldots \cdot 3 = 3^{11}$$

Thus the probability that the season ends in 7 wins, 2 losses, and 2 ties is

$$\frac{|E|}{|S|} = \frac{11!}{7!\,2!\,2!\,(3^{11})}$$

Suppose you randomly place 5 rooks on an 8×8 chessboard in non-attacking position. Suppose 2 of the rooks are yellow and three are blue.

Number of ways to place 2 yellow rooks and 3 blue rooks on an 8×8 chessboard where a yellow rook is in the first row and first column = $\frac{(7!)^2}{(3!)^3}$

Number of ways to place 2 yellow rooks and 3 blue rooks on an 8×8 chessboard = $\frac{(8!)^2}{(3!)^3(2!)}$

Thus the probability that a yellow rook is in the first row and first column is

$$\left(\frac{(7!)^2}{\cancel{(3!)^3}} \cdot \frac{\cancel{(3!)^3}(2!)}{(8!)^2} = \frac{2!}{64} = \frac{2}{64} = \frac{1}{32}\right)$$

What is the probability that a yellow rook is in the first row and second column.

$1/32$

$|A| = |B|$ iff there exists a bijection $f : A \to B$.

bij

1-1 correspondence

1-1 and onto

not bij

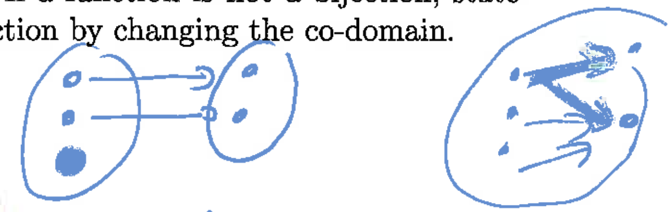
1:1, but not 1:1 correspondence

$f : A \to B$ is a bijection iff f is 1:1 and f is onto.

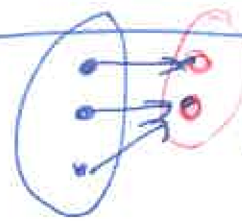
f is NOT a bijection iff f is not 1:1 OR f is not onto.

invertible

Determine if the following functions are bijections. If a function is not a bijection, state why and determine if you can create a bijective function by changing the co-domain.



not a function



not 1-1

1.) $f : R \to R,\ f(x) = x^2$

2.) $f : [0, \infty) \to R,\ f(x) = x^2$

3.) $f : [0, \infty) \to [0, \infty),\ f(x) = x^2$

4.) $f : R \to R,\ f(x) = x^3$

5.) $f : R \to R,\ f(x) = 2$

6.) $f : R \to R,\ f(x) = 8x + 2$

7.) $f : R \to R,\ f(x) = x^2 + 3x$

8.) $f : R \to R,\ f(x) = e^x$

9.) $f : R \to R,\ f(x) = x^4 + x^2$

10.) $f : R \to R,\ f(x) = sin(x)$

$f : A \to B$ is 1:1 iff $f(x_1) = f(x_2)$ implies $x_1 = x_2$.

$f : A \to B$ is 1:1 iff $x_1 \neq x_2$ implies $f(x_1) \neq f(x_2)$.

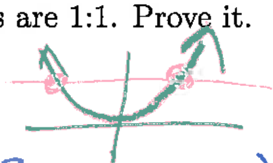
$f : A \to B$ is 1:1 iff for all $x_1 \neq x_2$, $f(x_1) \neq f(x_2)$.

$f : A \to B$ is NOT 1:1 iff there exists $x_1 \neq x_2$ such that $f(x_1) = f(x_2)$.

$1 \searrow 1$, $-1 \nearrow 1$

Determine if the following functions are 1:1. Prove it.

1.) $f : R \to R$, $f(x) = x^2$



$f(1) = 1 = f(-1)$ since $(1)^2 = (-1)^2$

2.) $f : [0, \infty) \to R$, $f(x) = x^2$

Suppose $f(x_1) = f(x_2) \Rightarrow x_1^2 = x_2^2 \Rightarrow x_1 = \pm\sqrt{x_2^2} = \pm|x_2|$

but $x_1, x_2 \geq 0 \Rightarrow x_1 = x_2$

3.) $f : [0, \infty) \to [0, \infty)$, $f(x) = x^2$

same as for #2

4.) $f : R \to R$, $f(x) = x^3$

5.) $f : R \to R$, $f(x) = 2$

6.) $f : R \to R$, $f(x) = 8x + 2$

7.) $f : R \to R$, $f(x) = x^2 + 3x$

8.) $f : R \to R$, $f(x) = e^x$

9.) $f : R \to R$, $f(x) = x^4 + x^2$

10.) $f : R \to R$, $f(x) = sin(x)$