2.6 Finite probability

Suppose $E \subset S$, then the probability of $E = P(E) = \frac{|E|}{|S|}$

S = sample space, E = events.

Note: we assume each outcome is equally likely

Ex: A football season consists of 11 games. What is the probability that the season ends in 7 wins, 2 losses, and 2 ties, IF it is equally likely that the football team wins, looses, or ties.

$P(\text{winning}) = 1/3 \quad P(\text{loosing}) = 1/3 \quad P(\text{tie}) = \frac{1}{3}$

The number of ways the season can end in 7 wins, 2 losses, and 2 ties is

\# of permutations of $\{7 \cdot w, 2 \cdot \ell, 2 \cdot t\}$

$$= \frac{11!}{7!\,2!\,2!}$$

The number of different ways in which the season can end is

\# of 11-permutations of $\{\infty \cdot w, \infty \cdot \ell, \infty \cdot t\}$ $\sim \{11 \cdot w, 11 \cdot \ell, 11 \cdot t\}$

$$= 3 \cdot 3 \cdot \ldots \cdot 3 = 3^{11}$$

Thus the probability that the season ends in 7 wins, 2 losses, and 2 ties is

$$\frac{|E|}{|S|} = \frac{11!}{7!\,2!\,2!\,(3^{11})}$$

Suppose you randomly place 5 rooks on an 8×8 chessboard in non-attacking position. Suppose 2 of the rooks are yellow and three are blue.

Number of ways to place 2 yellow rooks and 3 blue rooks on an 8×8 chessboard where a yellow rook is in the first row and first column = $\frac{(7!)^2}{(3!)^3}$

Number of ways to place 2 yellow rooks and 3 blue rooks on an 8×8 chessboard = $\frac{(8!)^2}{(3!)^3(2!)}$

Thus the probability that a yellow rook is in the first row and first column is

$$\frac{(7!)^2}{(\cancel{3!})^3} \cdot \frac{(\cancel{3!})^3(2!)}{(8!)^2} = \frac{2!}{64} = \frac{2}{64} = \boxed{\frac{1}{32}}$$

What is the probability that a yellow rook is in the first row and second column.

$1/32$