2.3 Combinations

\[ C(n, r) = \binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{P(n, r)}{r!} \]

\[ \sum_{i=0}^{n} \binom{n}{i} = 2^n \]

2.4 Combinations of Multisets

Thm 2.4.1: Let \( A = \{\infty \cdot 1, \infty \cdot 2, \ldots, \infty \cdot k\} \)

The number of \( r \) permutations of \( A = k^r \).

Ex: The number of 8-digit numbers with digits \( \{1, 2, 3, 4\} = 4^8 \)

Ex: The number of 9-digit numbers with digits \( \{0, 1, 2, 3\} = 3(4^8) \)

Thm 2.4.2: Let \( B = \{n_1 \cdot 1, n_2 \cdot 2, \ldots, n_k \cdot k\} \)

\[ n = n_1 + n_2 + \ldots + n_k \]

The number of permutations of \( B = \frac{n!}{n_1!n_2!\ldots n_k!} \).

\[ \frac{n!}{n_1!n_2!\ldots n_k!} = \binom{n}{n_1} \binom{n-n_1}{n_2} \ldots \binom{n-n_{k-1}}{n_k} \]

\# of permutations of \( \{n_1 \cdot 1, (n-n_1) \cdot 2\} = \frac{n!}{n_1!(n-n_1)!} = C(n, n_1) \)

Thm 2.4.3 If have \( n = n_1 + n_2 + \ldots + n_k \) different objects to be placed in \( k \) labeled boxes such that the box \( B_i \) contains \( n_i \) objects

\[ n! = n_1! \cdot n_2! \cdot \ldots \cdot n_k! \]
Ex 1) Suppose a traveling salesperson living in city H must visit five of the seven cities A, B, C, D, E, F, G. Find the number of different routes.

Note Ex 1 is a linear permutation, NOT a circular permutation.

Ex 2) Find the number of arrangements of six of eight letters A, B, C, D, E, F, G, H in a circle.

Ex 3) Find the number of arrangements of six of eight colors A, B, C, D, E, F, G, H in a bracelet.

Ex 4) Find the number of arrangements of six of eight letters A, B, C, D, E, F, G, H in a circle if the arrangement must include the letter H.

\[ \frac{7!}{5!} = \frac{7!}{2!} \]

Ex 5) How many different teams are possible if there must be 6 members on a team to be chosen from a group of 8 people.

Ex 6) How many different teams are possible if there must be at least one member on a team to be chosen from a group of 8 people.

Ex 7 (p. 45 bottom) The number of 2-combinations of the set \{1, 2, ..., n\} is

For each \(i\), the number of 2-combinations where \(i\) is the largest integer in the 2-combination is

Thus,
Example: How many 10-digit telephone numbers are there if

1.) there are no restrictions.

2.) the digits must all be distinct.

3.) The area code cannot begin with a 0 or 1 and must have a 0 or 1 in the middle.

Example: How many different seven-digit numbers can be constructed out of the digits 2, 4, 8, 8, 8, 8, 8?

\[
\frac{7!}{1! 5!} = \frac{7!}{5!} = 7 \cdot 6 \cdot 5 = 210
\]
2.3 Combinations

An r-combination of S is an r-element subset of S (ORDER DOES NOT MATTER).

\[ C(n, r) = \text{number of r-combinations of S where } |S| = n. \]

How many different math teams consisting of 4 people can be formed if there are 10 students from which to choose?

Thm: \[ C(n, r) = \binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{P(n,r)}{r!} \]

Cor: \[ C(n, r) = C(n, n-r) \]

Cor: \[ C(n, r) = C(n-1, r-1) + C(n-1, r) \]

Cor: Pascal’s Triangle.

Cor: \[ \sum_{i=0}^{n} \binom{n}{i} = 2^n \]

How many different proteins containing 10 amino acids can be formed if the protein contains 5 alanines (A), 3 leucines (L), and 2 serines (S)?

\[
\frac{10!}{5! \cdot 3! \cdot 2!}
\]