2.3 Combinations

$$C(n,r) = \binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{P(n,r)}{r!}$$

$$\Sigma_{i=0}^{n} \binom{n}{i} = 2^n$$

ALLOW REPEATS

2.4 Permutations of Multisets

Different ways of ordering same thing

Thm 2.4.1: Let $A = \{\infty \cdot 1, \infty \cdot 2, ..., \infty \cdot k\}$

unlimited repeats ✓

The number of r permutations of $A = k^r$.

Ex: The number of 8-digit numbers with digits $\{1, 2, 3, 4\} = 4^8$

$4 \cdot 4 \cdot \_ \_ \_ \_ \_ \cdot 4$

Ex: The number of 9-digit numbers with digits $\{0, 1, 2, 3\} = 3(4^8)$

can't have 0

Thm 2.4.2: Let $B = \{n_1 \cdot 1, n_2 \cdot 2, ..., n_k \cdot k\}$

limited repeats

$n = n_1 + n_2 + ... + n_k$

The number of permutations of $B = \frac{n!}{n_1! n_2! \cdots n_k!}$.

permutation of entire multiset

$$\frac{n!}{n_1! n_2! \cdots n_k!} = \binom{n}{n_1}\binom{n - n_1}{n_2} \cdots \binom{n - \Sigma_{i=1}^{k-1} n_i}{n_k}.$$

$\#$ of permutations of $\{n_1 \cdot 1, (n - n_1) \cdot 2\} = \frac{n!}{n_1!(n-n_1)!} = C(n, n_1)$

equality between 2.3 method for counting and 2.4 " " " when multiset contains only 2 different types of elements

Thm 2.4.3 If have $n = n_1 + n_2 + ... + n_k$ different objects to be placed in k labeled boxes such that the box B_i contains n_i objects

$$\frac{n!}{n_1! n_2! \ldots n_k!}$$

2.5 Combinations of Multisets

Let $A = \{\infty \cdot 1, \infty \cdot 2, ..., \infty \cdot k\}$

FRIDAY

The number of r combinations of $A = \binom{r+k-1}{r}$

Proof: An r combination is

Ex 1) Suppose a traveling salesperson living in city H must visit five of the the seven cities A, B, C, D, E, F, G. Find the number of different routes.

Note Ex 1 is a linear permutation, NOT a circular permutation.

Ex 2) Find the number of arrangements of six of eight letters A, B, C, D, E, F, G, H in a circle.

Ex 3) Find the number of arrangements of six of eight colors A, B, C, D, E, F, G, H in a bracelet.

HW hint

→ Ex 4) Find the number of arrangements of six of eight letters A, B, C, D, E, F, G, H in a circle if the arrangement must include the letter H.

$\frac{7!}{5!} = \frac{7!}{2!}$

H $\underline{7}$ $\underline{6}$ $\underline{5}$ $\underline{4}$ $\underline{3}$

Ex 5) How many different teams are possible if there must be 6 members on a team to be chosen from a group of 8 people.

Ex 6) How many different teams are possible if there must be at least one member on a team to be chosen from a group of 8 people.

Ex 7 (p. 45 bottom) The number of 2-combinations of the set $\{1, 2, ..., n\}$ is

For each i, the number of 2-combinations where i is the largest integer in the 2-combination is

Thus,

Example: How many 10-digit telephone numbers are there if

1.) there are no restrictions.

2.) the digits must all be distinct.

3.) The area code cannot begin with a 0 or 1 and must have a 0 or 1 in the middle.

Example: How many different seven-digit numbers can be constructed out of the digits 2, 4, 8, 8, 8, 8, 8?

$\{2, 4, 8, 8, 8, 8, 8\} = \{\text{"2"}, \text{"4"}, 5 \cdot \text{"8"}\}$

$\{2, 4, 8, 8, 8, 8, 8\} = 7$

$$\frac{7!}{1! \cdot 1! \, 5!} = \frac{7!}{5!}$$

Example: How many different seven-digit numbers can be constructed out of the digits 2, 2, 8, 8, 8, 8, 8?

$$\frac{7!}{2!\,5!}$$

2.3 Combinations

An r-*combination* of S is an r-element subset of S (ORDER DOES NOT MATTER).

$C(n,r)$ = number of r-combinations of S where $|S| = n$.

How many different math teams consisting of 4 people can be formed if there are 10 students from which to choose?

Thm: $C(n,r) = \binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{P(n,r)}{r!}$

Cor: $C(n,r) = C(n, n-r)$

Cor: $C(n,r) = C(n-1, r-1) + C(n-1, r)$

Cor: Pascal's Triangle.

Cor: $\Sigma_{i=0}^{n} \binom{n}{i} = 2^n$

How many different proteins containing 10 amino acids can be formed if the protein contains 5 alanines(A), 3 leucines (L), and 2 serines (S)?

$$\frac{10!}{5!\,3!\,2!}$$