18/28

2.2 Permutations: ← no repititions

Suppose $|S| = n$.

ORDER MATTERS

An *r-permutation* of S is an ordered arrangement of r of the n elements of S.

If $r = n$, then an r-permutation of S is a *permutation* of S.

$P(n,r)$ = number of r-permutations of S where $|S| = n$.

4 TA's need to be assigned to 4 different classes. How many different possible assignments are there?

$\underline{4}\ \underline{3}\ \underline{2}\ \underline{1} = 4! = P(4,4)$

4 classes need to be assigned a TA. There are 10 TAs. How many different possible assignments are there? TA assigned to at most 1 class

$\underline{10}\cdot\underline{9}\cdot\underline{8}\cdot\underline{7} = \frac{10!}{6!} = P(10,4)$

If $r > n$, then $P(n,r) = 0$

$P(0,0) = 1 \quad P(n,0) = 1 \quad P(n,1) = n \quad P(n,n) = n!$

$n! = n(n-1)(n-2)...(2)(1)$

$0! = 1$

Thm 2.2.1: $\boxed{\text{If } r \le n, \text{ then } P(n,r) = \frac{n!}{(n-r)!}}$

$\underset{1}{n}\cdot\underset{2}{(n-1)}\cdot\underset{3}{(n-2)}\cdots\underset{r}{n-(r-1)} = P(n,r) = \frac{n!}{(n-r)!}$

2.3 Combinations

An *r-combination* of S is an r-element subset of S (ORDER DOES NOT MATTER).

$C(n,r)$ = number of r-combinations of S where $|S| = n$.

choose 4 people to be on team OR choose 6 people to be not on team

EX: How many different math teams consisting of 4 people can be formed if there are 10 students from which to choose?

$C(10,4) = \frac{10!}{6!\,4!} = C(10,6)$

Thm: $C(n,r) = \binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{P(n,r)}{r!}$

Cor: $C(n,r) = C(n, n-r)$ ← check via algebra or combinatorially

Cor: $\boxed{C(n,r) = C(n-1, r-1) + C(n-1, r)}$

Cor: Pascal's Triangle.

Cor: $\Sigma_{i=0}^{n}\binom{n}{i} = 2^n$

sect. 2.4 problem

How many different proteins containing 10 amino acids can be formed if the protein contains 5 alanines(A), 3 leucines (L), and 2 serines (S)?

AAAAALLLSS

$\frac{P(10,10)}{5!\,3!\,2!} = \frac{10!}{5!\,3!\,2!}$

2.4: 2.2 + Division principle

all permutation
sect 2.2

Ex 1) Suppose a traveling salesperson living in city ~~H~~ must visit ~~five~~ six of the the ~~seven~~ eight cities A, B, C, D, E, F, G, H. Find the number of different routes.

order matters

permutations

$$P(8,6) = \frac{8!}{2!}$$

Note Ex 1 is a linear permutation, NOT a circular permutation.

Ex 2) Find the number of arrangements of six of eight letters A, B, C, D, E, F, G, H in a circle/bracelet

$$\frac{P(8,6)}{6}$$

since circular permutation

A ≠ ∀

Ex 3) Find the number of arrangements of six of eight colors A, B, C, D, E, F, G, H in a bracelet.

turn bracelet over

$$\frac{P(8,6)}{6 \cdot 2}$$

color A = color ∀

Ex 4) Find the number of arrangements of six of eight letters A, B, C, D, E, F, G, H in a circle if the arrangement must include the letter H.

$$P(7,5) = \frac{7!}{2!}$$

H 7 6 5 4 3

Ex 5) How many different teams are possible if there must be 6 members on a team to be chosen from a group of 8 people.

combinations 2.3

$$C(8,6) = \frac{8!}{2!\,6!} = \binom{8}{6}$$

Ex 6) How many different teams are possible if there must be at least one member on a team to be chosen from a group of 8 people.

$$\binom{8}{1}+\binom{8}{2}+\binom{8}{3}+\binom{8}{4}+\binom{8}{5}+\binom{8}{6}+\binom{8}{7}+\binom{8}{8} = 2^8-1$$

Ex 7 (p. 45 bottom) The number of 2-combinations of the set $\{1, 2, ..., n\}$ is $C(n,2) = \frac{n\cdot(n-1)}{2}$

For each i, the number of 2-combinations where i is the largest integer in the 2-combination is

Thus, $0 + 1 + 2 + \dots + n-1 = \frac{n(n-1)}{2}$

$\{\ ,1\}, \{1,2\}, \{^{1}_{2},3\}, \dots \{^{1}_{2}\vdots, n\}$

$n-1$

2.3 Combinations

$C(n,r) = \binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{P(n,r)}{r!}$

$\Sigma_{i=0}^{n} \binom{n}{i} = 2^n$

2.4 Permutations of Multisets

ALLOW REPEATS

Thm 2.4.1: Let $A = \{\infty \cdot 1, \infty \cdot 2, ..., \infty \cdot k\}$

The number of r permutations of $A = k^r$.

Ex: The number of 8-digit numbers with digits $\{1, 2, 3, 4\} = 4^8$

Ex: The number of 9-digit numbers with digits $\{0, 1, 2, 3\} = 3(4^8)$

Thm 2.4.2: Let $B = \{n_1 \cdot 1, n_2 \cdot 2, ..., n_k \cdot k\}$

$n = n_1 + n_2 + ... + n_k$

The number of permutations of $B = \frac{n!}{n_1! n_2! \cdots n_k!}$.

$\frac{n!}{n_1! n_2! \cdots n_k!} = \binom{n}{n_1} \binom{n - n_1}{n_2} \cdots \binom{n - \Sigma_{i=1}^{k-1} n_i}{n_k}$.

\# of permutations of $\{n_1 \cdot 1, (n - n_1) \cdot 2\} = \frac{n!}{n_1!(n-n_1)!} = C(n, n_1)$

Thm 2.4.3 If have $n = n_1 + n_2 + ... + n_k$ different objects to be placed in k labeled boxes such that the box B_i contains n_i objects

2.~~4~~5 Combinations of Multisets

Let $A = \{\infty \cdot 1, \infty \cdot 2, ..., \infty \cdot k\}$

The number of r combinations of $A = \binom{r + k - 1}{r}$

Proof: An r combination is