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2.2 Permutations: ← no repititions

Suppose $|S| = n$.

ORDER MATTERS

An *r-permutation* of S is an ordered arrangement of r of the n elements of S.

If $r = n$, then an r-permutation of S is a *permutation* of S.

$P(n, r)$ = number of r-permutations of S where $|S| = n$.

4 TA's need to be assigned to 4 different classes. How many different possible assignments are there?

$\underline{4}\ \underline{3}\ \underline{2}\ \underline{1} = 4! = P(4,4)$

4 classes need to be assigned a TA. There are 10 TAs. How many different possible assignments are there? TA assigned to at most 1 class

$\underline{10}\cdot\underline{9}\cdot\underline{8}\cdot\underline{7} = \frac{10!}{6!} = P(10,4)$

If $r > n$, then $P(n, r) = 0$

$P(0, 0) = 1$ $\quad P(n, 0) = 1$ $\quad P(n, 1) = n$ $\quad P(n, n) = n!$

$n! = n(n-1)(n-2)...(2)(1)$

$0! = 1$

Thm 2.2.1: $\boxed{\text{If } r \leq n, \text{ then } P(n, r) = \frac{n!}{(n-r)!}}$

$n\cdot(n-1)\cdot(n-2)\cdot\ldots\cdot n-(r-1) = P(n,r) = \frac{n!}{(n-r)!}$

1 2 3 r

2.3 Combinations

An *r-combination* of S is an r-element subset of S (ORDER DOES NOT MATTER).

$C(n, r)$ = number of r-combinations of S where $|S| = n$.

choose 4 people to be on team OR choose 6 people to be not on team

EX: How many different math teams consisting of 4 people can be formed if there are 10 students from which to choose?

$C(10,4) = \frac{10!}{6!\,4!} = C(10,6)$

Thm: $C(n, r) = \binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{P(n,r)}{r!}$

Cor: $C(n, r) = C(n, n-r)$ ← check via algebra

Cor: $C(n, r) = C(n-1, r-1) + C(n-1, r)$

Cor: Pascal's Triangle.

Cor: $\Sigma_{i=0}^{n} \binom{n}{i} = 2^n$

Sect. 2.4 problem

How many different proteins containing 10 amino acids can be formed if the protein contains 5 alanines(A), 3 leucines (L), and 2 serines (S)?

AAAAALLLSS

$\frac{P(10,10)}{5!\,3!\,2!} = \frac{10!}{5!\,3!\,2!}$

2.4 : 2.2 + Division principle

Example: How many 10-digit telephone numbers are there if

1.) there are no restrictions.

2.) the digits must all be distinct.

3.) The area code cannot begin with a 0 or 1 and must have a 0 or 1 in the middle.

Section 2.4 Division principle

$$\frac{7!}{5!} = \frac{P(7,7)}{5!}$$

Example: How many different seven-digit numbers can be constructed out of the digits 2, 4, 8, 8, 8, 8, 8?

Section 2.2 method

$$\underline{7} \cdot \underline{6} = P(7,2)$$

7 spots

Example: How many different seven-digit numbers can be constructed out of the digits 2, 2, 8, 8, 8, 8, 8?

see 2.4

$$\frac{P(7,2)}{2!}$$

all permutions of 1rst "2" and 2nd "2"

in terms of placement

Example A: How many numbers between 100 and 1000 have distinct digits.

$$\underline{9} \cdot \underline{9} \cdot \underline{8}$$

Example B: How many odd numbers between 100 and 1000 have distinct digits.

$$\underline{8} \cdot \underline{8} \cdot \underline{5}$$

Example C: How many even numbers between 100 and 1000 have distinct digits.

method 1:

method 2:

method 3: $9 \cdot 9 \cdot 8 - 8 \cdot 8 \cdot 5$

six eight

Ex 1) Suppose a traveling salesperson living in city ~~H~~ X must visit ~~five~~ of the the ~~seven~~ cities A, B, C, D, E, F, G, H. Find the number of different routes.

order matters   permutations $P(8,6) = \frac{8!}{2!}$

Note Ex 1 is a linear permutation, NOT a circular permutation.

Ex 2) Find the number of arrangements of six of eight letters A, B, C, D, E, F, G, H in a circle./bracelet

$\frac{P(8,6)}{6}$ $A \neq \forall$

Ex 3) Find the number of arrangements of six of eight colors A, B, C, D, E, F, G, H in a bracelet.

turn bracelet over   color A = color $\forall$

Ex 4) Find the number of arrangements of six of eight letters A, B, C, D, E, F, G, H in a circle if the arrangement must include the letter H.

Ex 5) How many different teams are possible if there must be 6 members on a team to be chosen from a group of 8 people.

Ex 6) How many different teams are possible if there must be at least one member on a team to be chosen from a group of 8 people.

Ex 7 (p. 45 bottom) The number of 2-combinations of the set $\{1, 2, ..., n\}$ is

For each i, the number of 2-combinations where i is the largest integer in the 2-combination is

Thus,