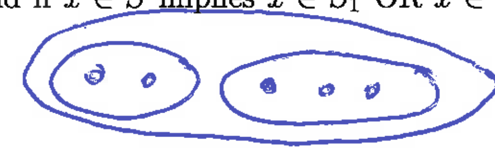
A *partition* of a set S is a collection of subsets S_i of S such that $S = \cup S_i$ and $S_i \cap S_j = \emptyset$ for all $i \neq j$.

Addition Principle: If $S = S_1 \cup S_2$ and $S_1 \cap S_2 = \emptyset$, then $|S| = |S_1| + |S_2|$.

If $S_1 \cap S_2 = \emptyset$ and if $x \in S$ implies $x \in S_1$ OR $x \in S_2$, then $|S| = |S_1| + |S_2|$.



$2+3=5$

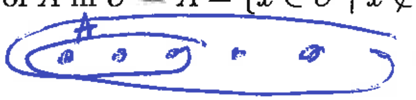
Multiplication Principle: If $S = S_1 \times S_2$, then $|S| = |S_1||S_2|$. $\blacksquare$

$x = (a, b) \in S$ implies $a \in S_1$ AND $b \in S_2$, ~~then~~ $|S| = |S_1||S_2|$.

$2 \times 3 = 6$

$\{(1,1), (1,2), (2,1), (2,2)$
$(3,1,\ 3,2)\}$

Subtraction Principle: Suppose $A \subset U$. Let the complement of A in $U = \overline{A} = \{x \in U \mid x \notin A\}$. Then $|A| = |U| - |\overline{A}|$.



$|\overline{A}| = 5 - 3$

Division Principle: Suppose $S = \cup_{i=1}^{k} S_i$. If $|S_i| = n$ $\forall$i, then $k = \frac{|S|}{n}$.

$3 = \frac{|6|}{2}$

$\frac{6}{3} = 2$

$|S_i| = 2$

Counting Problems:

1.) Order matters (ordered arrangements or selections)

1a.) no repeats allowed ← sec 2.2 Permutation

1b.) (limited) repeats allowed ← 2.4

2.) Order does not matter (unordered arrangements or selections)

2a.) no repeats allowed 2.3 Combination

2b.) (limited) repeats allowed ← 2.5

Defn: A *multiset* is a collection of objects were repeats are allowed.

Set: $\{a, a, b, b, b\} = \{a, b\}$

Multiset: $\{a, a, b, b, b\} = \{2 \cdot a, 3 \cdot b\}$

Subsets: Suppose a set B has n elements (i.e., $|B| = n$). The number of subsets of B is

2^n

Suppose a symbol can be either a number between 0 and 9 or a letter. How many are symbols there?

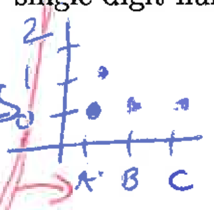
Addition principle

$\{0-9\} \cup \{A-Z\}$

$10 + 26 = \boxed{36}$

How many sequences consisting of one letter followed by one single digit number (0 - 9) are possible?

permutation ORDER MATTERS



$\boxed{\text{Mult principle}}$

order matters

$\underset{\text{letter}}{\underline{26}} \times \underset{\text{number}}{\underline{10}} = 260$

How many different license plates are possible if 3 letters followed by 3 numbers are used?

$\underbrace{\underline{26} \times \underline{26} \times \underline{26}}_{\{A-Z\}} \times \underbrace{\underline{10} \times \underline{10} \times \underline{10}}_{\{0-9\}} = \boxed{26^3 \times 10^3}$

REPEATS ALLOWED

How many different license plates are possible if 3 letters followed by 3 numbers are used and the license plate starts with a vowel if and only if the plate contains exactly one vowel?

See chalk board notes

$\phi = \{\ \}$

ϕ, $\{a\}$, $\{b\}$, $\{c\}$, $\{d\}$,

Subsets $\{a,b\}$, $\{a,c\}$, ... $\{a,b,c,d\}$

Suppose a set A has four elements (i.e., the cardinality of $A = |A| = 4$)

$\{a, b, c, d\}$

The number of subsets of A is

$\underset{\text{a or no a in subset}}{\underline{2}} \times \underset{\text{b or no b in subset}}{\underline{2}} \times \underset{\text{c or no c}}{\underline{2}} \times \underset{\text{d or no d}}{\underline{2}} = 2^4$

The number of nonempty subsets of A is

A pizza parlor offers 4 different toppings (sausage, onions, chicken, walnuts). How many different types of pizzas can one order?

Suppose a set B has n elements (i.e., $|B| = n$). The number of subsets of B is

2^n.

Suppose a symbol can be either a number between 0 and 9 or a letter. How many are symbols there?

How many sequences consisting of one letter followed by one single digit number (0 - 9) are possible?

How many different license plates are possible if 3 letters followed by 3 numbers are used?

How many different license plates are possible if 3 letters followed by 3 numbers are used and the license plate starts with a vowel if and only if the plate contains exactly one vowel?

$\{a, c, d\}$

$\{\ \} \leftrightarrow$ don't include a, b, c, d

Subsets

$\{a, b, c, d\} \leftrightarrow$ include a, b, c, d

Suppose a set A has four elements (i.e., the cardinality of $A = |A| = 4$) $\{a, b, c, d\}$

The number of subsets of A is

$2 \; 2 \; 2 \; 2 = 2^4$

include a / don't include a

include b or no b

can no c

d or no d

The number of nonempty subsets of A is

$2^4 - 1$

toss out $\emptyset$ from our list of subsets

A pizza parlor offers 4 different toppings (sausage, onions, chicken, walnuts). How many different types of pizzas can one order?

2^4

Suppose a set B has n elements (i.e., $|B| = n$). The number of subsets of B is

$2 \cdot 2 \cdot \ldots \cdot 2 = 2^n$

permutions
order
matters

Example: How many 10-digit telephone numbers are there if

1.) there are no restrictions. repeats allowed

$\underline{10} \cdot \underline{10} \cdots \underline{10} = 10^{10}$

2.) the digits must all be distinct. no repeats

$\underline{10} \cdot \underline{9} \cdot \underline{8} \cdot \underline{7} \cdot \underline{6} \cdot \underline{5} \cdot \underline{4} \cdot \underline{3} \cdot \underline{2} \cdot \underline{1} = 10!$

3.) The area code cannot begin with a 0 or 1 and must have a 0 or 1 in the middle. repeats allowed

$\underline{8} \cdot \underline{2} \cdot \underline{10} \cdot \underline{10} \cdots \underline{10} = 8 \cdot 2 \cdot 10^8$

Example: How many different seven-digit numbers can be constructed out of the digits 2, 4, 8, 8, 8, 8, 8?

$\underline{7} \cdot \underline{6} = 42$

# of choices for placing 2

# of choices for placing 4 if 2 already placed

Example: How many different seven-digit numbers can be constructed out of the digits 2, 2, 8, 8, 8, 8, 8?

$2 \cdot 4$

42

$\frac{42}{2} = 21$

permutation

$100 \leq x \leq 1000$

Example A: How many numbers between 100 and 1000 have distinct digits.

$\underline{9} \cdot \underline{9} \cdot \underline{8}$

{1-9} {0,..9} but not first

Example B: How many odd numbers between 100 and 1000 have distinct digits.

$\underline{8} \cdot \underline{8} \cdot \underline{5}$

{1-9} {0-9} {1, 3, 5, 7, 9}

2 3 1

So filled in blanks from most to least restrictive something

Example C: How many even numbers between 100 and 1000 have distinct digits.

method 1: $9 \cdot 8 \cdot 1 + 8 \cdot 8 \cdot 4$

=

method 2: $9 \cdot 8 \cdot 5 - 1 \cdot 8 \cdot 4$

$\underline{\quad}\ \underline{\quad}\ \underline{5}$

{1-9} {0-9} {0, 2, 4, 6, 8}

EX = 8488288, 8288488

8288288