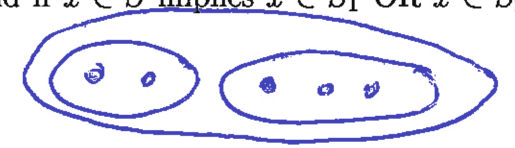
2.1 Basic Counting

A *partition* of a set S is a collection of subsets S_i of S such that $S = \cup S_i$ and $S_i \cap S_j = \emptyset$ for all $i \neq j$.

Addition Principle: If $S = S_1 \cup S_2$ and $S_1 \cap S_2 = \emptyset$, then $|S| = |S_1| + |S_2|$.

If $S_1 \cap S_2 = \emptyset$ and if $x \in S$ implies $x \in S_1$ OR $x \in S_2$, then $|S| = |S_1| + |S_2|$.



$2+3=5$

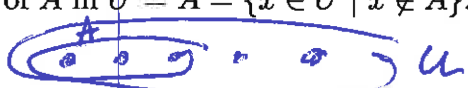
Multiplication Principle: If $S = S_1 \times S_2$, then $|S| = |S_1||S_2|$.

$x = (a, b) \in S$ implies $a \in S_1$ AND $b \in S_2$, ~~then~~ $|S| = |S_1||S_2|$.

$2 \times 3 = 6$

$\{(1,1), (1,2), (2,1), (2,2), (3,1,\ 3,2)\}$

Subtraction Principle: Suppose $A \subset U$. Let the complement of A in $U = \overline{A} = \{x \in U \mid x \notin A\}$. Then $|A| = |U| - |\overline{A}|$.



$|\overline{A}| = 5 - 3$

Division Principle: Suppose $S = \cup_{i=1}^{k} S_i$. If $|S_i| = n\ \forall$i, then $k = \frac{|S|}{n}$.

$3 = \frac{|6|}{2}$

$\frac{6}{3} = 2$

$|S_i| = 2$

Counting Problems:

1.) Order matters (ordered arrangements or selections)

1a.) no repeats allowed

1b.) (limited) repeats allowed

2.) Order does not matter (unordered arrangements or selections)

2a.) no repeats allowed

2b.) (limited) repeats allowed

Defn: A *multiset* is a collection of objects were repeats are allowed.

Set: $\{a, a, b, b, b\} = \{a, b\}$

Multiset: $\{a, a, b, b, b\} = \{2 \cdot a, 3 \cdot b\}$

Subsets: Suppose a set B has n elements (i.e., $|B| = n$). The number of subsets of B is

2^n

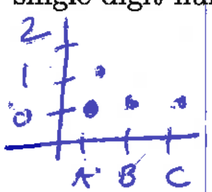
Suppose a symbol can be either a number between 0 and 9 or a letter. How many are symbols there?

Addition principle

$\{0-9\} \cup \{A-Z\}$

$10 + 26 = \boxed{36}$

How many sequences consisting of one letter followed by one single digit number (0 - 9) are possible?



$\boxed{\text{Mult principle}}$

order matters

$\underset{\text{letter}}{\underline{26}} \times \underset{\text{number}}{\underline{10}} = 260$

How many different license plates are possible if 3 letters followed by 3 numbers are used?

$\underbrace{\underline{26} \times \underline{26} \times \underline{26}}_{\{A-Z\}} \times \underbrace{\underline{10} \times \underline{10} \times \underline{10}}_{\{0-9\}} = \boxed{26^3 \times 10^3}$

How many different license plates are possible if 3 letters followed by 3 numbers are used and the license plate starts with a vowel if and only if the plate contains exactly one vowel?

See chalk board notes

$\phi = \{\ \}$

ϕ, $\{a\}$, $\{b\}$, $\{c\}$, $\{d\}$, $\{a,b\}$, $\{a,c\}$, ... $\{a,b,c,d\}$

Subsets

Suppose a set A has four elements (i.e., the cardinality of $A = |A| = 4$)

$\{a, b, c, d\}$

The number of subsets of A is

$\underset{\text{a or no a in subset}}{\underline{2}} \times \underset{\text{b or no b in subset}}{\underline{2}} \times \underset{\text{c or no c}}{\underline{2}} \times \underset{\text{d or no d}}{\underline{2}} = 2^4$

The number of nonempty subsets of A is

A pizza parlor offers 4 different toppings (sausage, onions, chicken, walnuts). How many different types of pizzas can one order?

Suppose a set B has n elements (i.e., $|B| = n$). The number of subsets of B is

2^n