Math 150 Final Exam
December 17, 2009

Choose 6 from the following 9 problems. Circle your choices: 1 2 3 4 5 6 7 8 9
You may do more than 6 problems in which case your unchosen problems can replace your lowest one or two problems at 4/5 the value as discussed in class.

1.) Use a generating function to find the number of combinations of $\{3 \cdot a, 1 \cdot b, \infty \cdot c\}$ which have exactly 10 elements.

2.) Use inclusion-exclusion to to find the number of combinations of $\{3 \cdot a, 1 \cdot b, \infty \cdot c\}$ which have exactly 10 elements.

3.) Use Theorem 14.2.3 to determine the number of nonequivalent colorings of the corners of a rectangle that is not a square with the colors red and blue. Do the same with p colors.

4.) Solve the following homogeneous recurrence relation: $h_n = 4h_{n-1} + 5h_{n-2}$.

5a.) Use the binomial theorem to prove that $2^n = \Sigma_{k=0}^{n} \binom{n}{k}$.

5b.) Use a combinatorial argument to prove that $2^n = \Sigma_{k=0}^{n} \binom{n}{k}$.

6a.) Suppose S is a set of n integers. Show that given any $d < n$, there exists $x, y \in S$ such that $x \neq y$ and d divides $x - y$.

6b.) Show that the Ramsey number $r(3,3) > 5$.

7a.) $\binom{1.6}{3} =$ ________________________

7b.) Let $f = \begin{pmatrix} 1234 \\ 1342 \end{pmatrix}$.

Let $\mathbf{c} : \{1,2,3,4\} \to \{red, blue\}$, $\mathbf{c}(i) = \begin{cases} red & i = 0 \bmod 2 \\ blue & \text{otherwise} \end{cases}$

$f^{-1} =$ ________________________ $f * \mathbf{c} =$ ________________________

8a.) The inversion sequence for the permutation 1324 is ______________________________

8b.) Define the relation $\leq$ on $\mathcal{R} \times \mathcal{R}$ by $(a, b) \leq (c, d)$ if $a < c$ or if $a = c$, $b \leq d$. Show the relation $\leq$ on $\mathcal{R} \times \mathcal{R}$ is anti-symmetric.

9.) Let $A_1 = \{1, 2\}$, $A_2 = \{3, 5, 6\}$, $A_2 = \{4\}$ and let $X = \{1, 2, 3, 4, 5, 6\}$. Define a relation R on X by xRy if there exists A_i such that $x, y \in A_i$.

Draw R as a subset of $X \times X$. Determine which of the following properties hold for R (Prove it).

Is R reflexive?

Is R irreflexive?

Is R symmetric?

Is R antisymmetric?

Is R transitive?

Is R an equivalence relation?

If so, use R to partition X into its equivalence classes.

Is R a partial order?