Math 150 Exam 2
October 30, 2009

Choose 6 from the following 8 problems. Circle your choices: 1 2 3 4 5 6 7 8
You may do more than 6 problems in which case one of your two unchosen problems can replace your lowest problem at 4/5 the value as discussed in class.

1.) $\binom{2.3}{4} =$

2a.) State the axiom of choice (you can give either a formal or informal definition).

2b.) State a cyclic Gray code of order 3.

3.) Let $\mathcal{P} = \{P_\alpha \mid \alpha \in A\}$ be a partition of X. Define a relation $\sim$ on X by $x \sim y$ if and only if there exists $P_\alpha \in \mathcal{P}$ such that $x, y \in P_\alpha$. Show that $\sim$ is an equivalence relation.

4.) Let $\mathcal{Z}$ be the set of integers. Define the equivalence relation $\sim$ on $\mathcal{Z}$ by $x \sim y$ if and only if $5|(x-y)(xy-1)$. Show that $\sim$ is reflexive and symmetric. Use $\sim$ to partition $\mathcal{Z}$ into its equivalence classes. Make sure the sets in your partition are pairwise disjoint.

5.) Let $X = \{1, 2, 3, 4\}$. Define the relation R on X by xRy if and only if $3|(2x - y)$. Draw R as a subset of $X \times X$. Determine which of the following properties hold for R (Prove it).

Is R reflexive?

Is R irreflexive?

Is R symmetric?

Is R antisymmetric?

Is R transitive?

6.) Determine the number of 10-combinations of $\{5 \cdot a, 5 \cdot b, 5 \cdot c\}$.

7.) Prove that $(x+y+z)^n = \Sigma \begin{pmatrix} & n & \\ n_1 & n_2 & n_3 \end{pmatrix} x^{n_1} y^{n_2} z^{n_3}$.

8a.) Use the binomial theorem to prove that $2^n = \Sigma_{k=0}^{n} \binom{n}{k}$.

8b.) Generalize to find the sum $\Sigma_{k=0}^{n} \binom{n}{k} r^k$.