1.) Show that every sequence $a_1, a_2, ..., a_{n^2+1}$ contains either an increasing or decreasing subsequence of length $n + 1$.

2.) The Ramsey number $r(3, 3) =$ _______. Prove your answer.

3.) Is the intersection $R \cap S$ of two equivalence relations $R$ and $S$ on a set $X$ always an equivalence relation on $X$? Is the union $R \cup S$ of two equivalence relations $R$ and $S$ on a set $X$ always an equivalence relation on $X$? Prove your answer.

4.) Find the number of integral solutions to the equation $x_1 + x_2 + x_3 + x_4 = 60$ such that $0 \leq x_1 \leq 10$, $1 \leq x_2 \leq 5$, $x_3 \geq -2$, and $x_4 \geq 4$.

5.) Let $D_n$ be the number of derangements of $\{1, 2, ..., n\}$. Determine a formula for $D_n$. Prove your answer.

6.) Solve the recurrence relation $h_n = 2h_{n-1} + 3^n$ with initial value $h_0 = 4$

7a.) Determine the generating function for the number $h_n$ of $n$-combinations of fruit consisting of apples, oranges, bananas, pears, and kiwis in which there are an odd number of apples, the number of oranges is a multiple of 4, the number of bananas is at most 3, the number of pears is 0 or 1, and there are at least 2 kiwis.

7b.) Find a formula for $h_n$.

8a.) Find the number of partitions of 6 distinguishable objects into 3 nonempty distinguishable boxes.

8b.) Find the difference table for $h_n = n^2 + 1$

8c.) $\sum_{k=0}^{n} h_k =$ ____________
9a.) Find the number of subsets of \( \{1, 2, 3, \ldots, 10\} \).

9b.) Find the number of subsets of \( \{1, 2, 3, \ldots, 10\} \) which have exactly 8 elements.

9c.) Find the number of permutations of \( \{1, 2, 3, \ldots, 10\} \) which have exactly 8 elements.

9d.) Find the number of permutations of \( \{3 \cdot a, 4 \cdot b, 1 \cdot c\} \) which have exactly 8 elements.

9e.) Find the number of partitions of 25 indistinguishable objects into 10 distinguishable boxes.

10a.) Expand \((x - 2y)^6\) using the binomial theorem.

10b.) What is the coefficient of \(x^4y^3z^2\) in the expansion of \((x - y + 3z)^9\)?:

10c.) What is the coefficient of \(x^3y^3z^2\) in the expansion of \((x - y + 3z)^9\)?:

10d.) The inversion sequence for the permutation 615423 is

10e.) The permutation corresponding to the inversion sequence 5, 1, 3, 2, 1, 0 is