Choose 7 from the following 9 problems. Circle your choices: 1 2 3 4 5 6 7 8 9
You may do more than 7 problems in which case your two unchosen problems can replace your lowest one or two problems at 2/3 the value as discussed in class.

1.) \( \text{P}(10, 7) = (10)(9)(8)(7)(6)(5)(4) \)
\[
\text{C}(10, 7) = \binom{10}{7} = \frac{10!}{7!3!} = \frac{(10)(9)(8)}{(3)(2)(1)} = (10)(3)(4) = 120
\]
The inversion sequence for the permutation 15243 is 0, 1, 2, 1, 0
The permutation corresponding to the inversion sequence 3, 0, 2, 1, 0 is 2, 5, 4, 1, 3

2.) \( r(9, 2) = 9 \)
\( r(3, 3) = 6 \)

Before \( \{x_{13}, x_{12}, x_7, x_1\} \): section4.3
After \( \{x_{13}, x_{12}, x_7, x_1\} \): section4.3
Before \( \{2, 8, 13, 14\} \): section4.3
After \( \{2, 8, 13, 14\} \): section4.3

3.) In how many ways can 9 indistinguishable rooks be placed on a 20-by-20 chessboard so that no rook can attack another rook?

\[
\frac{20!}{9!(11)!} \quad \frac{20!}{9!11!} \]

In how many ways can 9 rooks be placed on a 20-by-20 chessboard so that no rook can attack another rook if no two rooks have the same color?

\[
\frac{20!}{9!(11)!} - \frac{20!}{9!9!} \]

4.) How many different circular permutations can be made using using 30 beads if you have 20 green beads, 9 blue beads and 1 red beads?

\[
\frac{29!}{20!9!} \]

5.) How many sets of 3 numbers each can be formed from the numbers \( \{1, 2, 3, ..., 50\} \) if no two consecutive numbers are to be in a set?

Suppose we think of the 50 numbers as 50 sticks. The number of ways of removing 3 sticks such that no two are consecutive is the same as the number of integral solutions to \( x_1 + x_2 + x_3 + x_4 = 47 \) where \( x_1, x_4 \geq 0 \) and \( x_2, x_3 \geq 1 \). This is the same as the number of solutions to \( x_1 + y_2 + 1 + y_3 + 1 + x_4 = 47 \) where \( x_1, x_4 \geq 0, y_2 = x_2 - 1 \geq 1 - 1 = 0, \)
\[ y_3 = x_3 - 1 \geq 1 - 1 = 0. \] This is the same as the number of solutions to \( x_1 + y_2 + y_3 + x_4 = 45 \) where \( x_1, x_4, y_2, y_3 \geq 0. \)

Hence by thm 3.5.1, the answer is \( \binom{45 + 4 - 1}{45} = \binom{48}{45} = \frac{48(47)(46)}{6} \)

6.) Use the pigeonhole principle to prove that in a group of \( n \) people where \( n > 1 \), there are at least 2 people who have the same number of acquaintances. State where you use the pigeonhole principle.

Number the people 1 through \( n \). We will assume that all acquaintances are mutual. We will also assume that you can’t be your own acquaintance. Thus if person \( i \) has \( k_i \) acquaintances among the group of \( n \) people, \( k_i \in \{0, \ldots, n - 1\} \).

Case 1: There exists someone who knows everyone else. Then \( k_i \in \{1, \ldots, n - 1\} \) for \( i = 1, \ldots, n \). Thus by the pigeonhole principle, there exists \( i \neq j \) such that \( k_i = k_j \).

Case 2: There does not exist someone who knows everyone else. Then \( k_i \in \{0, \ldots, n - 2\} \) for \( i = 1, \ldots, n \). Thus by the pigeonhole principle, there exists \( i \neq j \) such that \( k_i = k_j \).

7.) section 4.5

8.) section 4.5

9.) Use a combinatorial argument to prove \( \sum_{k=0}^{n} \binom{n}{k} \binom{n}{n-k} = \binom{2n}{n} \)

\( \binom{2n}{n} \) = the number of ways to choose \( n \) elements from \( \{1, \ldots, 2n\} \).

\( \binom{n}{k} \) = the number of ways to choose \( k \) elements from \( \{1, \ldots, n\} \).

\( \binom{n}{n-k} \) = the number of ways to choose \( n-k \) elements from \( \{n+1, \ldots, 2n\} \).

Suppose \( A \) is an \( n \)-element subset of \( \{1, \ldots, 2n\} \). Let \( k = |A \cap \{1, \ldots, n\}| \).

Thus to choose an \( n \)-element subset of \( \{1, \ldots, 2n\} \), we can first fix \( k \) and choose \( k \) elements from \( \{1, \ldots, n\} \) and \( n-k \) elements from \( \{n+1, \ldots, 2n\} \). For a fixed \( k \), the number of ways of choosing \( k \) elements from \( \{1, \ldots, n\} \) and \( n-k \) elements from \( \{n+1, \ldots, 2n\} \) is \( \binom{n}{k} \binom{n}{n-k} \). To get all \( n \) element subset of \( \{1, \ldots, 2n\} \), we must do this for \( k = 0, \ldots, n \). Thus the number of ways to choose \( n \) elements from \( \{1, \ldots, 2n\} = \sum_{k=0}^{n} \binom{n}{k} \binom{n}{n-k} \).