www.geometrygames.org/TorusGames

Equivalence class \([a] = \{ x \mid x \sim a \}\)

\(\mathcal{P} = \{P_\alpha \mid \alpha \in A\}\) is a partition of \(X\) iff
\(X = \bigcup_{P_\alpha \in \mathcal{P}} P_\alpha, \ P_\alpha \neq \emptyset \ \forall \alpha, \text{ and } P_\alpha \cap P_\beta = \emptyset\)

Suppose \(X = \bigcup_{\alpha \in B} R_\alpha\) and \(R_\alpha \neq \emptyset \ \forall \alpha, \text{ and } R_\alpha \cap R_\beta \neq \emptyset\)
implies \(R_\alpha = R_\beta\). Then \(\mathcal{R} = \{R_\alpha \mid \alpha \in B\}\) is a partition of \(X\)

Suppose \(a, b \in \mathbb{Z} - \{0\}\. \ a \sim b \text{ if } ab > 0\)

\([4] =\)

\([-2] =\)

Ex: \(\mathbb{Z} - \{0\} = \bigcup_{n \in \mathbb{Z} - \{0\}} [n]\)

\[= \bigcup_{n \in \mathbb{Z} - \{0\}} [2n] = (\bigcup_{n=-1}^{-\infty} [2n]) \cup (\bigcup_{n=1}^{\infty} [2n])\]

Thm 4.5.3: If \(\sim\) is an equivalence relation on \(X\), then
\(\{[x_\alpha] \mid x_\alpha \in X\}\) is a partition of \(X\).

If \(\mathcal{P} = \{P_\alpha \mid \alpha \in A\}\) is a partition of \(X\), then
\(x \sim y \text{ iff } \exists P_\alpha \text{ such that } x, y \in P_\alpha\) is an equivalence relation.