Suppose a multiset consisting of integers between 0 and 5 inclusive of size \( k \) must contain the following:

- even number of 0’s: \( x^0 + x^2 + x^4 + \ldots = \frac{1}{1-x^2} \)
- odd number of 1’s: \( x^1 + x^3 + x^5 + \ldots = \frac{x}{1-x^2} \)
- three or four 2’s: \( x^3 + x^4 = x^3(1 + x) \)
- the number of 3’s is a multiple of five: \( x^0 + x^5 + x^{10} + \ldots = \frac{1}{1-x^5} \)
- between zero to four (inclusive) 4’s: \( x^0 + x^1 + x^2 + + x^3 + x^4 = \frac{1-x^5}{1-x} \)
- zero or one 5: \( x^0 + x^1 = 1 + x \)

\[ g(x) = (x^0 + x^2 + x^4 + \ldots)(x^1 + x^3 + x^5 + \ldots)(x^3 + x^4) \]
\[ (x^0 + x^5 + x^{10} + \ldots)(x^0 + x^1 + x^2 + + x^3 + x^4)(x^0 + x) \]

\[ = \left( \frac{1}{1-x^2} \right) \left( \frac{x}{1-x^2} \right) x^3(1 + x) \left( \frac{1}{1-x^5} \right) \left( \frac{1-x^5}{1-x} \right) (1 + x) \]

\[ = \frac{x^4}{(1-x)^3} = x^4 \sum_{k=0}^{\infty} \binom{3+k}{k} - 1 \]
\[ x^k = \sum_{k=0}^{\infty} \frac{(k+2)(k+1)}{2} x^{k+4} \]

Find the number of multisets of size \( n \).

Find the number of multisets of size 100.
Determine the generating function for \( h_n = \) the number of ways to make \( n \) cents using pennies, nickels, dimes, and quarters.

Note \( h_n = \) the number of nonnegative integral solutions to

\[
e_1 + 5e_2 + 10e_3 + 25e_4 = n
\]

Let \( f_1 = e_1, f_2 = 5e_2, f_3 = 10e_3, f_4 = 25e_4, \)

Then \( h_n = \) the number of nonnegative integral solutions to

\[
f_1 + f_2 + f_3 + f_4 = n
\]

where \( f_1 \) is a nonnegative integer

\( f_2 \) is a multiple of 5

\( f_3 \) is a multiple of 10

\( f_4 \) is a multiple of 25

Hence the generating function for \( h_n \) is