Suppose $f : N \to M$ is smooth. Then $d_p f : T_p(N) \to T_p(M)$.

If df_p is $1-1$, for all $p \in N$, then f is called an *immersion.*
I.e., f is an immersion iff f has rank n

If df_p is onto for all $p \in N$, then f is called a *submersion.*
I.e., f is an submersion iff f has rank m

Defn. Suppose $f : M \to N$ is smooth.

$p \in M$ is a *critical point* and $f(p)$ is a *critical value* if rank $df_p < n$.

If $p \in M$ is not a critical point, then it is a *regular point.*

If $q \in N$ is not a *critical value*, then it is a *regular value.*

Note: $q \in N$ is a regular value iff $f^{-1}(q) = \emptyset$ or $\forall p \in f^{-1}(q)$, $df_p = n$.

Defn: If K is a submanifold of M, then K has the subspace topology.
Also, for all $p \in K$, there exists a chart (ϕ, U) for M such that $\phi(U) = \Pi_1^m(-\epsilon, \epsilon)$ and $\phi(U \cap K) = \Pi_1^k(-\epsilon, \epsilon) \times \Pi_{k+1}^m\{0\}$, then
Also, $(\phi_{U \cap K}, U \cap K)$, $\phi|_{U \cap K} : U \cap K \to \Pi_1^k(-\epsilon, \epsilon)$ is a chart for K.
The collection of such charts form a pre-atlas for K.

Thm 2.3.13: Let q be a regular value of f: $M \to N$. Then either $f^{-1}(q) = \emptyset$ or $f^{-1}(q)$ is an $(m-n)$-submanifold of M.

Defn: Suppose f: $M \to N$ is a $1-1$ immersion, and suppose f: $M \to f(M)$ is a homeomorphism, where $f(M) \subset N$ has the relative topology. Then f is an *embedding*, and $f(M)$ is an embedded submanifold.

Thm. $f : M \to N$ embedding implies $f(M)$ is a submanifold of N.