Suppose \( f : N \to M \). Then \( d_p f : T_p(N) \to T_p(M) \).

If \( df_p \) is \( 1 - 1 \), for all \( p \in N \), then \( f \) is called an \textit{immersion}.

I.e., \( f \) is an immersion iff \( f \) has rank \( n \)

If \( df_p \) is onto for all \( p \in N \), then \( f \) is called a \textit{submersion}.

I.e., \( f \) is an submersion iff \( f \) has rank \( m \)

Defn. Suppose \( f : M \to N \) is smooth.

\( p \in M \) is a \textit{critical point} and \( f(p) \) is a \textit{critical value} if rank \( df_p < n \).

If \( p \in M \) is not a critical point, then it is a \textit{regular point}.

If \( q \in N \) is not a \textit{critical value}, then it is a \textit{regular value}.

Note: \( q \in N \) is a regular value iff \( f^{-1}(q) = \emptyset \) or \( \forall p \in f^{-1}(q), \ df_p = n \).

Defn: \( K \) is a \textit{m-submanifold} of \( N \) if \( \forall q \in K \subset N, \exists g^{smooth} : V^{open} \subset N \to \mathbb{R}^{n-m}, \ q \in V \) such that

1.) \( g \) is smooth

2.) \( K \cap V = g^{-1}(0) \) and

3.) rank \( d_p g = n - m \)

Defn: Suppose \( f : M \to N \) is a \( 1 - 1 \) immersion, and suppose \( f : M \to f(M) \) is a homeomorphism, where \( f(M) \subset N \) has the relative topology. Then \( f \) is an \textit{embedding}, and \( f(M) \) is an embedded submanifold.

Thm. \( f : M \to N \) embedding implies \( f(M) \) is a submanifold of \( N \).

Thm 2.3.13: Let \( q \) be a regular value of \( f : M \to N \). Then either \( f^{-1}(q) = \emptyset \) or \( f^{-1}(q) \) is an \( (m - n) \)-submanifold of \( M \).