

Defn: Suppose $f : W \rightarrow N$ where $W \subset M$ and N are smooth manifolds. f is *smooth* if for all $p \in W$, \exists charts (ϕ, U) and (ψ, V) and such that $p \in U$, $f(p) \in V$, $f(U) \subset V$ and $\psi \circ f \circ \phi^{-1}$ is smooth.

0.) If $f : W \rightarrow N$ is smooth, then given charts (ϕ, U) and (ψ, V) such that $f(U) \subset V$, then $\psi \circ f \circ \phi^{-1}$ is smooth.

1.) If $f : W \rightarrow N$ is smooth, then f is continuous.

2.) If $f : W \rightarrow N$ is smooth, $V^{open} \subset W$, then $f : V \rightarrow N$ is smooth.

3.) $f : W \rightarrow N$, $W = \cup U_\alpha^{open}$, and $f : U_\alpha \rightarrow N$ smooth for all α , then $f : W \rightarrow N$ is smooth.

4.) If f, g smooth, $f \circ g$ is smooth.

Defn: $f : M \rightarrow N$ is a *diffeomorphism* if f is a homeomorphism and if f and f^{-1} are smooth. M and N are *diffeomorphic* if there exists a diffeomorphism $f : M \rightarrow N$.

Prop 1.2.9: Let M and N be smooth manifolds, and let $\{U_\alpha, \phi_\alpha\}$ be the atlas for N . Suppose $f : M \rightarrow N$ is a diffeomorphism. Then $\{f^{-1}(U_\alpha), \phi_\alpha \circ f\}$ is the atlas for M .