

## 133 Practice Exam

This is a 40 minute exam. Work two questions from this practice exam. Indicate on the front page of your exam which problems you want graded. You may use results about functions in euclidean space (chain rule, implicit or inverse function theorems for example) or "big theorems" (such as Sard's theorem, the regular value theorem) provided you give the precise statement.

Due 5pm Thursday July 24 (slide it under my office door, B1H MLH).

- 1.) Suppose  $M, N$  are smooth manifolds and  $f : M \rightarrow N$  is smooth. Show that  $X = \{(p, f(p)) \mid p \in M\}$  is a regular submanifold of  $M \times N$ .
  
- 2.) Let  $G : (\frac{\pi}{2}, \frac{3\pi}{2}) \rightarrow \mathbb{R}^2$ ,  $G(t) = (\cos(t), \sin(2t))$ . Show that  $G$  is a 1-1 immersion, but not an embedding.
  
- 3.) Let  $G$  be a Lie group. Show that  $G$  admits a never-zero vector field.