Math 132 Midterm
August 2004

[20-40pts] Answer all of the following 6 questions. You do not need to prove your answer for these first 6 questions. Note for the first 5 questions, you are working in $\mathcal{Z}$, the set of integers, (under various topologies including the subspace topology) and NOT the set of real numbers.

1.) Let $X = \mathcal{Z}$, then the set of limit points of $(0,3) \cap X$ in X, $[(0,3) \cap X]' =$

2.) Let $X = \mathcal{Z}$, then the closure of $(0,3) \cap X$ in X, $\overline{(0,3) \cap X} =$

3.) Let $X = \mathcal{Z}$, then the interior of $(0,3) \cap X$ in X, $[(0,3) \cap X]^o =$

4.) Suppose $X = \mathcal{Z}$ has the indiscrete topology, then the set of limit points of $(0,3) \cap X$ in X, $[(0,3) \cup X]' =$

5.) Suppose $X = \mathcal{Z}$ has the indiscrete topology, then the interior of $(0,3) \cap X$ in X, $[(0,3) \cap X]^o =$

6.) If d is the discrete metric on $\mathcal{R}$, the set of real numbers, then

$B_d(0,1) =$ ____________ $B_d(0,2) =$ ____________

[60-80pts] Prove 4 of the following 6. You may do more than these 4 problems for possible partial credit as discussed in class.

Your best three answers are: ___________________

Your fourth best answer is: ______

1. Let $X = \{(x, y) \in R^2 \mid x = 0 \text{ or } y = 0\}$ and let $W = \{(x, y) \in R^2 \mid y = 0\}$ where X and W are subspaces of R^2. Define $f : X \to W$ by $f(x, y) = (x, 0)$.

   Is f continuous? Is f an open map? Is f a closed map. In each case, prove your answer.

2. The **fixed point set** F **of a function** $f : X \to X$ is the set $F = \{x \in X : f(x) = x\}$. Show that if X is Hausdorff and f is continuous then F is a closed subset of X.

3. Let $f : X \to Y$ be a bijective continuous function. If X is compact and Y is Hausdorff, prove that f is a homeomorphism. Give a specific example to show that f need not be a homeomorphism if X is not compact. Also give a specific example to show that f need not be a homeomorphism if Y is not Hausdorff.

4. Show that X is compact if and only if every collection $\mathcal{C}$ of closed sets in X having the finite intersection property, the intersection $\cap_{C\in\mathcal{C}} C$ of all the elements of $\mathcal{C}$ is nonempty.

5. Consider the product, uniform, and box topologies on R^ω.

   In which topologies are the following functions from the set of real numbers to R^ω continuous (prove your answer):

   $f(x) = (x, x^2, x^3, ...)$

   $g(x) = (x, x, x, ...)$

   $k(x) = (0, 0, 0, ...)$

6. Let R^∞ be the subset of R^ω consisting of all sequences that are eventually zero. What is the closure of R^∞ in R^ω in the uniform topology. What is the closure of R^∞ in R^ω in the box topology. Prove your answer.