section 18: 13) Suppose \( A \subset X \). Let \( f : A \rightarrow Y \) be continuous where \( Y \) is \( T_2 \). If \( g_i : \overline{A} \rightarrow Y \) is continuous for \( i = 1, 2 \) and if \( g_1|_A = g_2|_A = f \), then \( g_1 = g_2 \)

Pf: Suppose \( g_1 \neq g_2 \). Then \( \exists x \in \overline{A} - A \) such that \( g_1(x) \neq g_2(x) \). \( Y \) \( T_2 \) implies \( \exists V_1, V_2 \) open in \( Y \) such that \( g_i(x) \in V_i \) and \( V_1 \cap V_2 = \emptyset \)

\[ x \in g_i^{-1}(V_i) \] implies \( x \in g_1^{-1}(V_1) \cap g_2^{-1}(V_2) \). \( g_i \) continuous implies \( g_i^{-1}(V_i) \) open in \( \overline{A} \) and hence \( g_1^{-1}(V_1) \cap g_2^{-1}(V_2) \) open in \( \overline{A} \). Thus there exists \( U \) open in \( X \) such that \( g_i^{-1}(V_i) \cap g_2^{-1}(V_2) = U \cap \overline{A} \). Note \( x \in U \).

\[ x \in \overline{A} - A \] implies \( x \in \overline{A}' \). Thus \( \exists a \in U \cap A - \{x\} \subset g_1^{-1}(V_1) \cap g_2^{-1}(V_2) \). But \( f(a) = g_i(a) \in g_i(g_i^{-1}(V_i)) \subset V_i \). But \( V_1 \cap V_2 = \emptyset \).

NOTE: It may have been simpler to WLOG assume \( X = \overline{A} \), but you must state why this WLOG works.