Final Exam in Topology

December 15, 2010

Instructions. Do at least four problems. This is a closed book examination, you should have no books or paper of your own. Please do your work on the paper provided. Clearly number your pages corresponding to the problem you are working. When you start a new problem, start a new page; only write on one side of the paper. Make a cover page and indicate clearly which four problems you want graded.

Always justify your answers unless explicitly instructed otherwise. You may use theorems if the problem is not a step in proving that theorem, but you need to state any theorems you use carefully.

Please indicate here which four problems you want to have graded:

A1 A2 A3 A4 A5 A6

1. Every metrizable space is normal.

2. Prove or disprove: The finite product of separable spaces is separable (prove all topology theorems used).

3. a.) Define locally finite.

   b.) Suppose X is locally finite, then $\overline{\cup_{\alpha \in A} C_\alpha} = \cup_{\alpha \in A} \overline{C_\alpha}$.

   c.) Is it true in general that $\overline{\cup_{\alpha \in A} C_\alpha} = \cup_{\alpha \in A} \overline{C_\alpha}$?

4. a.) If X is a 2-manifold, then $\forall x \in X$, there exists an open neighborhood U of x which is homeomorphic to $(-1, 1) \times (-1, 1)$.

   b.) If X is a 2-manifold, then $\forall x \in X$, there exists an open neighborhood U of x which is homeomorphic to R^2.

   c.) If X is a 2-manifold, then $\forall x \in X$, there exists a closed neighborhood U of x which is homeomorphic to $[-1, 1] \times [-1, 1]$.

   d.) If X is a 2-manifold, then X is locally compact.

5. If X is a compact m-manifold, then X can be imbedded in $\mathbb{R}^N$ for some positive integer N. Hint: You can use the fact that given a finite open covering of X, there exists a partition of unity dominated by that cover.

6. Suppose Y^X has the uniform topology with uniform metric $\overline{\rho}$. Suppose $f : X \to Y$ is a function.

   a.) Prove or disprove: $B_{\overline{\rho}}(f, \epsilon) = \Pi_{\alpha \in X} B_d(f(\alpha), \epsilon)$ [hint for counter-example, consider $\mathbb{R}^\omega$].

   b.) Prove or disprove: $B_{\overline{\rho}}(f, \epsilon) = \cup_{\delta < \epsilon} \Pi_{\alpha \in X} B_d(f(\alpha), \delta)$.

   c.) If $X = Y = \mathbb{R}$, give an example of a function in $B_{\overline{\rho}}(x^2, 1)$.