2.) Suppose $\mathbb{R}^\omega$ has the uniform topology with uniform metric $\overline{p}$.

[4] 2a.) $\overline{p}(x,y) =$ ____________________________

[4] 2b.) $B_{\overline{p}}(0,2) =$ ____________________________

[4] 2c.) Let $x_n = (\frac{1}{n}, \frac{1}{n}, \frac{1}{n}, ...) \in \mathbb{R}^\omega$. Let $A = \{x_i \mid i \in \mathbb{Z}_+\}$. Then $A^o =$ ____________________________ .

[18] 3.) Circle $T$ for true and $F$ for false. If a statement is false, show that the statement is false by providing a counter-example. You do not need to prove that your example is a counter-example.

3a.) If $A$ is a compact subspace of $X$, then $A$ is closed in $X$. T F

3b.) Let $A$ be a connected subspace of $X$. If $A \subset B \subset \overline{A}$, then $B$ is connected. T F

3c.) Let $A$ be a path connected subspace of $X$. If $A \subset B \subset \overline{A}$, then $B$ is path connected. T F
[60] Prove 2 of the following 5. Clearly indicate your choices. You may do a third problem for extra credit.

First two choices: _____________

Third choice (extra credit): _____________

1. Compact Hausdorff implies $T_3$.

2. Define an equivalence relation on $\mathbb{R}^1$ by $x \sim y$ if $x - y \in \mathbb{Z}$. Let $X/\sim$ be the corresponding quotient space. It is homeomorphic to a familiar space. What is it? [Hint: set $g(x) = e^{2\pi x}$]

3. Let $H$ be a subspace of the topological group $(G, \cdot)$. Show that if $H$ is also a subgroup of $G$, then both $H$ and $\overline{H}$ are topological groups. Hint: Recall that $H$ is a subgroup of the group $G$ if and only if it is nonempty and closed under products and inverses.

4. Let $x_n = (\frac{1}{n}, \frac{1}{n}, \frac{1}{n}, \ldots) \in \mathbb{R}^\omega$. Let $A = \{x_i \mid i \in \mathbb{Z}_+\}$. If $\mathbb{R}^\omega$ has the uniform topology, determine $\overline{A}$.

5. Suppose $X$ is locally compact and $f : X \to Y$ is a continuous, surjective, open map. Then $f(X)$ is locally compact.