1.) Define: Basis for a topology

2.) Suppose $\mathbb{R}$ = the set of real numbers has the standard topology. Calculate the following in \((0, \infty)\) where \((0, \infty) \subset \mathbb{R}\) has the subspace topology:

\[(0, 1)^{\circ} = \quad \quad \quad (0, 1) = \quad \quad \quad (0, 1)' = \quad \quad \quad\]

3.) The following two statements are false. Show that the statements are false by providing counter-examples. Very briefly explain your counter-examples.

3a.) Suppose $A \subset Y \subset X$. Let $A^\circ$ denote the interior of $A$ in $X$ and let $Int_Y A =$ interior of $A$ in $Y$. Then $Int_Y A = A^\circ \cap Y$.

3b.) $\overline{A \cap B} = \overline{A} \cap \overline{B}$. 
Prove 2 of the following 4. **Clearly indicate your TWO choices.** You may do more than 2 problems in which case I may substitute one of your unchosen problems for one of your two choices (with a penalty) if it improves your grade.

1. (2\(\mathbb{Z}\), +) is a topological group where (2\(\mathbb{Z}\), +) is the even integers with the operation of addition.

2. \(f_i : X_i \to Y_i\) continuous implies \(F : X_1 \times X_2 \to Y_1 \times Y_2\), \(F(x_1, x_2) = (f_1(x_1), f_2(x_2))\) is continuous.

3. Let \(\mathbb{R}\) have the topology \(T_{ray} = \{(r, \infty) \mid r \in \mathbb{R}\}\). Find \((0, 1)\) in this topology. Fully prove.

4. Suppose \(T_\alpha\) is a topology for all \(\alpha \in A\)
   a.) Prove or disprove: \(\bigcup_{\alpha \in A} T_\alpha\) is a topology.
   b.) Prove or disprove: \(\bigcap_{\alpha \in A} T_\alpha\) is a topology.