Theorem 1. The subspace of a regular space is regular - Theorem 31.2 from Munkres.

Proof. Let $Y$ be a subspace of a regular space $X$. Then one-point sets are closed in $Y$.
Let $x$ be any point in $Y$ and let $B$ be a closed subset of $Y$ disjoint from $x$. Then $\overline{B} \cap Y = B$, where $\overline{B}$ is the closure of $B$ in $X$.
Thus $x \notin \overline{B}$, so by using the definition of regularity of $X$, we can choose disjoint open sets $U$ and $V$ of $X$ containing $x$ and $\overline{B}$, respectively.
Then $U \cap Y$ and $V \cap Y$ are disjoint open sets in $Y$ containing $x$ and $B$, respectively. □