The continuous image of a Lindelöf space is also Lindelöf

Let $f : X \to f(X)$ be a continuous map and $\mathcal{A}$ be any covering of $f(X)$. Then $\{f^{-1}(A) | A \in \mathcal{A}\}$ is a collection of sets covering $X$; these sets are open in $X$ by the continuity of $f$. Then, since $X$ is a Lindelöf space, $\exists$ a countable subcover $\{f^{-1}(A_n)\}$ of $\{f^{-1}(A) | A \in \mathcal{A}\}$. Then $\{A_n\}$ is a countable subcover of the original cover $\mathcal{A}$. Thus, $f(X)$ is Lindelöf.