22M:132: Topology Exam 2
Nov. 21, 2008

[10] 1.) Definition: X is locally compact if

[30] 3.) Circle T for true and F for false. If a statement is false, show that the statement is false by providing a counter-example. You do not need to prove that your example is a counter-example.

3a.) A continuous bijective function $f : X \to Y$ is a homeomorphism if X is T_2 and Y is compact.
T F

3b.) A subspace of a second countable space is second countable. T F

3c.) A subspace of a limit point compact space is limit point compact T F

3d.) Connected implies path connected. T F

[60] Prove 2 of the following 4. Clearly indicate your choices. Note **R** is the set of real numbers. You may do a third problem for extra credit.

First two choices: ____________

Third choice (extra credit): ____________

1. Let $x \sim y$ if $x - y \in \mathcal{Z}$ where $\mathcal{Z}$ is the set of integers. Let $\mathbf{R}^* = \mathbf{R}/\sim$ with the quotient topology. Identify $\mathbf{R}^*$. Prove.

2. A metric space is normal.

3. **(a.)** If $f : [0,1] \to [0,1]$ is continuous, then f has a fixed point (i.e., there exists x such that $f(x) = x$). **(b.)** Show that $g : [0,1) \to [0,1)$ need not have a fixed point even if g is continuous.