22M:132: Topology Exam 1 Sept 30, 2008

[10] 1.) Definition: The point  $x \in X$  is a limit point of A if

[12] 2.) Suppose  $\mathbf{R}$  = the set of real numbers has the standard topology. Let  $\mathbf{Q}$  = the set of rational numbers. Calculate the following in  $\mathbf{R}$ :

 $\mathbf{Q}^{\,o} =$ \_\_\_\_\_  $\mathbf{\overline{Q}} =$ \_\_\_\_\_  $\mathbf{Q}^{\,\prime} =$ \_\_\_\_\_

3.) The following two statements are false. Show that the statements are false by providing counterexamples. You do not need to explain your counter-examples.

[9] 3a.) If  $x_n \in A$ , then there exists a unique point  $x \in \overline{A}$  such that  $x_n \to x$ .

[9] 3b.) If  $f: X \to Y$  is continuous, then  $\overline{f(A)} \subset f(\overline{A})$ .

- [60] Prove 2 of the following 3. Clearly indicate your choices.
  - 1. The product of two Hausdorff spaces is Hausdorff.
  - 2. Let Y be a subspace of X. If  $A \subset Y$ , then  $Cl_Y(A) = \overline{A} \cap Y$ .
  - 3. Show that  $D((x_1, y_1), (x_2, y_2)) = min\{|x_1 x_2|, \frac{|y_1 y_2|}{2}\}$  is a metric on  $\mathbb{R}^2$  where  $(x_i, y_i) \in \mathbb{R}^2$ . Show that this metric generates the standard topology on  $\mathbb{R}^2$ .