Is a compact Hausdorff space locally compact? Yes.
Is a compact Hausdorff space locally compact Hausdorff? Yes.

**Claim.** All compact spaces are locally compact.

**Definition.** A space $X$ is said to be **locally compact at** $x$ if there is some compact subspace $C$ of $X$ that contains a neighborhood of $x$. If $X$ is locally compact at each of its points, $X$ is said simply to be **locally compact.** -Munkres, 2nd Ed., §29

Since $X$ is a compact space, choose $C := X$. The same choice holds for each $x \in X$, giving us that $X$ is locally compact.